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VOLUME LX NUMBER 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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Yerkes Observatory of the University of Chicago

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SEPTEMBER 1924

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VOLUME LX SEPTEMBER 1924 NUMBER 2

THE RELATIVE SIZES OF THE KERNELS OF TEN ELECTRONS OF *Na*, *Mg*, *Al*, AND *Si* DERIVED FROM THE HOMOLOGOUS SPECTRA, AND THEIR RELATION TO THE *L* X-RAY LEVELS OF THE LIGHT ATOMS

By LOUIS A. TURNER

ABSTRACT

The relative sizes of the kernels of ten electrons of Na, Mg, Al, and Si are computed from the *Na* I, *Mg* II, *Al* III, and *Si* IV spectra. Assuming 1 for *Na*, they are *Mg*, 0.740; *Al*, 0.575; *Si*, 0.472. These ratios indicate that the effective nuclear charge for the outer orbits of the kernel is equal to the true nuclear charge *minus* a screening constant, the same for all four atoms, and approximately the same as the screening constant derivable from the *L* X-ray levels of the light atoms.

In a recent paper[1] it has been shown that some of the terms in the homologous spectra of the alkali metal atoms, corresponding to orbits of higher azimuthal quantum number, are of magnitudes in agreement with the assumption that the field of force in which the electron circulates is geometrically similar, but different in size, for those atoms. A. Fowler in his paper[2] on the spectrum of trebly ionized silicon has compared the terms of that spectrum with those of the spectrum of doubly ionized aluminum, of singly ionized magnesium, and of neutral sodium, and has shown that there are great similarities between them. This result is that predicted by Bohr's theory of atomic structure, since all of these spectra are

[1] Louis A. Turner, *Astrophysical Journal*, **58**, 176, 1923.

[2] *Proceedings of the Royal Society*, A, **103**, 413, 1923.

thought to arise from systems of the same number of electrons having the same quantum numbers. The differences seem to be in accord with the theory of similar fields of force.

In the previous paper it was shown that the distance of the valence electron from the nucleus when its velocity is perpendicular to the radius vector is given by the expression

$$r=\frac{pNa_0}{\nu}\left(1\pm\sqrt{1-\frac{k^2\nu}{p^2N}}\right) \tag{1}$$

where N is the Rydberg constant, ν is the wave number of the term, k is the azimuthal quantum number, pe is the charge on the equivalent point nucleus $\left(\text{i.e., potential energy}=-p\frac{e^2}{r}\right)$, and a_0 is the radius of the electron's orbit in the normal hydrogen atom, as given by Bohr's theory. There is only one value of r for circular orbits, so that for such orbits the expression under the radical sign has to vanish and p can be computed. With this value of p, r can then be computed. Assuming that $3d$, $4b$, and $5f$ terms correspond to circular orbits of azimuthal quantum numbers 3, 4, and 5, respectively, Table I is computed from the data given by Fowler.

TABLE I

	ν	p	r	log r	Z	log $\{10^4(10+p-Z)\}$
Na	12276.2	1.0037	$8.9670a_0$	0.953	11	1.568
$Mg+$	49976.8	2.0210	4.4532	0.645	12	2.322
$Al++$	113484	3.0516	2.9493	0.470	13	2.713
$Si+++$	203705	4.0885	2.2013	0.343	14	2.941
Na	6860.4	1.0004	$15.994a_0$	1.204	11	0.602
$Mg+$	27467.6	2.0018	7.993	0.903	12	1.255
$Al++$	61827.3	3.0032	5.337	0.727	13	1.505
$Si+++$	109923	4.0045	3.995	0.602	14	1.653
$Si+++$	70213	4.00055	$6.249a_0$	0.796	14	0.740

p can be considered as the sum of two coincident point charges, one, the nuclear charge, $+Ze$, where Z is the atomic number, and another charge representing the effective charge of the electron system $p_e e$. $p_e=p-Z$. It is seen that for all cases this effective charge is slightly less than ten times *minus* e, being less by the amount of the decimal part of p which equals $10+p-Z$ or $10+p_e$. These decimal parts of p can be used to describe the field of the

kernel, a method convenient for the present purpose. Ten is the number of electrons in each of these kernels.

It was also shown in the previous paper that p has the same value at corresponding points in the fields of geometrically similar kernels of the same net charge. The same argument applies here for values of p_e. Hence, if the fields are geometrically similar, the curves giving p_e as a function of r for the different atoms should transform into each other by multiplication of the values of r for one curve by the ratio of the sizes of the two kernels. Actually, it is easier

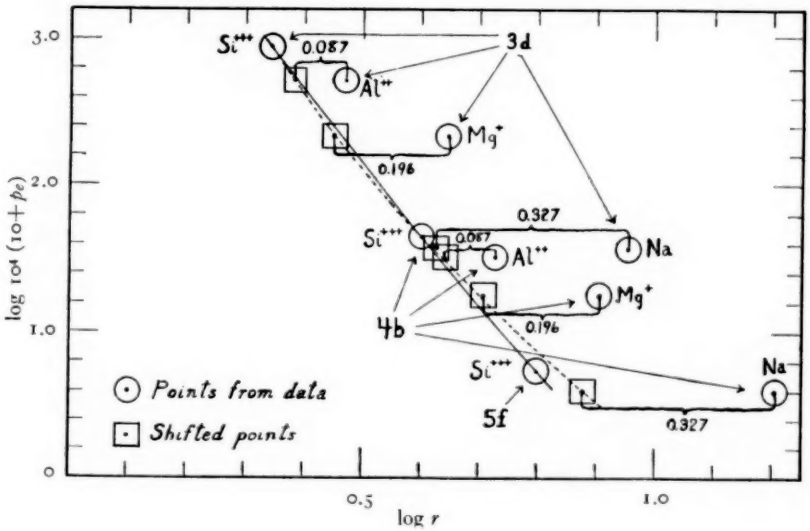

FIG. 1

to find the proper factor by plotting values of log r and accomplishing the multiplication by a shift parallel to the log r axis. Also, it is more convenient to plot values of log $10^4(10+p_e)$, instead of values of p_e. Figure 1 shows the points plotted in this way.

A curve is drawn through the three points for *Si*. The horizontal shifts, log f, necessary to bring the other six points on this curve are given in Table II.

If, however, we disregard the $5f$ point for *Si*, the dotted curve can be drawn through the other two *Si* points so that the other six points can be brought on to it by horizontal shifts such that those for the two points of each element will be quite accurately equal. That is,

parallel curves can be drawn for the four elements, in agreement with the theory. The $5f$ term point for *Si* which would lie on the dotted curve would correspond to a value of ν, 10 wave-number units greater than the ν used. The calculation of the term values for this *Si* spectrum by Fowler depended on three differences between successive b levels, as given by differences between measured lines, and the assumption that these b levels are accurately representable by the Ritz formula. The actual values of the four b levels concerned were found by computation on this basis by the method of least squares. Thus it may well be that the values so determined are

TABLE II

	log f	Mean
Al++	−0.082, −0.094	−0.088
Mg+	.180, .216	.198
Na	−0.332, −0.379	−0.356

TABLE III

	Log f	f	Size ($Na=1$)
Si+++	0.000	1.00	0.472
Al++	.087	1.22	.575
Mg+	.196	1.57	0.740
Na	0.327	2.12	1.000

in error by a few wave-number units. An error of ten units would be an error of approximately one part in five thousand for the b levels computed. It should be noted that such an error would not change the accuracy of representation of spectrum lines by the system, for it would be a small additive error in all terms which would cancel upon taking the difference between two of them to find the wave number of any line. The method of plotting used here greatly exaggerates possible errors in terms which yield values of p but slightly greater than an integer, so that a small error in all the *Si* terms would have great effect on the $5f$ point, and practically no effect on the other two *Si* points.

The shifts for the dotted curve and the corresponding ratios, f's, of the sizes of the kernels to that of *Si* are given in Table III. The sizes, assuming $Na=1$, are also given.

In a discussion of the theoretical interpretations of the X-ray spectrum levels Bohr and Coster[1] point out that for a hydrogen-like atom the energy and size of an elliptical orbit are given by the expressions

$$W = Nh\frac{Z^2}{n^2}, \qquad 2a = \frac{e^2}{Nh} \cdot \frac{n^2}{Z} \tag{2}$$

(W = energy, n = total quantum number, a = semi-major axis of the ellipse), that for electron orbits in atoms in general we may define the effective nuclear charge, Z^*, and effective quantum number, n^*, by similar equations

$$W = Nh\frac{Z^{*2}}{n^{*2}}, \qquad 2a = \frac{e^2}{Nh} \cdot \frac{n^{*2}}{Z^*}, \tag{3}$$

and that for the inner orbits the effective quantum numbers differ but little from the actual quantum numbers. The effective nuclear charge is less than the actual one by the amount of a screening constant, which is almost the same for atoms of nearly equal numbers of electrons. These considerations should apply to the orbits of the kernels of the atoms considered here, and we should expect the relative sizes of these kernels to be those of the orbits of the outer electrons of the kernels. If so, these relative kernel sizes should be inversely proportional to the atomic number ***minus*** a screening constant. A curve with atomic numbers for abscissae and reciprocals of the sizes for ordinates should be a straight line with an intercept equal to the screening constant. Figure 2 shows the curve so obtained. The intercept is 8.3. Similarly, the square roots of the energies of the outer orbits of the kernels should bear a linear relation to the atomic numbers, and give the same value of the screening constant. The L X-ray levels presumably correspond to the orbits of electrons in these kernels, and the L-term values give the energies of the orbits. Using the values of $\sqrt{\nu/N}$ given by Bohr and Coster for the $L_{II} - L_{III}$ levels of the light atoms, a portion of a Moseley curve is drawn in Figure 2. Its intercept on the Z axis is 7.3. Thus, there is but fair agreement between the two screening constants. Some difference would be expected, however, because the energy of the orbit which we should use here is the energy necessary to remove the electron without changing

[1] N. Bohr and D. Coster, *Zeits. für Phys.*, **12**, 342, 1923.

any of the others, whereas the L X-ray level corresponds to the energy necessary to put the atom in a state with one of these electrons missing and the others in correspondingly changed orbits. Also, the assumption has been made that the ratio of the major axes of the two orbits would equal the ratio of the average effective sizes of the orbits which give the apparent sizes of the kernels. This is not necessarily accurately true.

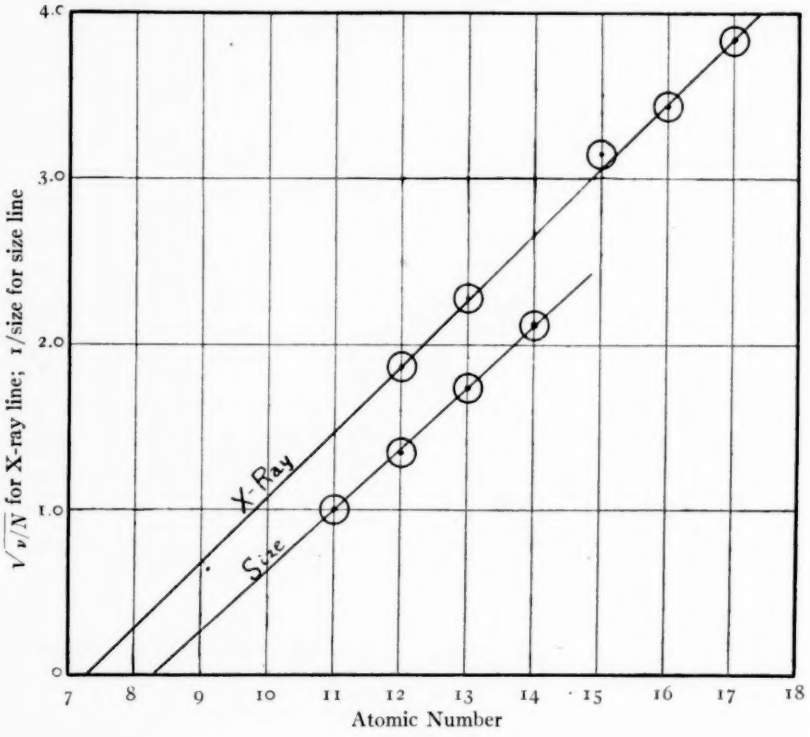

FIG. 2

Thus it has been possible to determine from optical data the relative sizes of kernels of ten electrons of neutral sodium, singly ionized magnesium, doubly ionized aluminum, and trebly ionized silicon, and to show that they give a screening constant approximately that predicted by use of the L X-ray term values. The kernels shrink with increasing nuclear charge, as would be expected.

PALMER PHYSICAL LABORATORY
PRINCETON, NEW JERSEY
January 1924

RADIOMETER MEASUREMENTS OF STELLAR ENERGY SPECTRA[1]

By C. G. ABBOT

ABSTRACT

Apparatus.—The author employed the new type Nichols radiometer, 0.5 mm wide, with the coudé arrangement of the 100-inch telescope on Mount Wilson. A mirror system of collimation and image-forming, with a 60° flint-glass prism, produced a spectrum of 43-mm dispersion from $\lambda = 0.437\ \mu$ to $\lambda = 2.224\ \mu$.

Energy spectra observed.—Radiometer deflections at various points in the spectra of the sun and nine bright stars were observed visually on a scale at 5 meters from the radiometer. The stars observed were β Orionis, α Lyrae, α Canis Maj., α Canis Min., α Aurigae, α Tauri, β Pegasi, α Orionis, and α Herculis. Alpha Orionis gave maximum spectral deflections of 50 mm.

Reduction to normal scale.—Atmospheric transmission having been allowed for, as determined for Mount Wilson by solar work, the results were reduced to normal energy outside the atmosphere, through the intermediary of the Smithsonian solar observations of 1920 and 1922. The results are given in Table V and Figure 4.

The results amplified by visual and photographic work.—Being of small weight for wave-lengths less than $0.55\ \mu$, the radiometric results were supplemented by the visual and photographic photometry of Wilsing, Scheiner, Münch, and Rosenberg. Very satisfactory agreement appeared in overlapping portions of the spectrum.

Comparison with black-body curves.—The combined results having been plotted, they were compared with the energy curve of the perfect radiator, to determine the absolute centigrade temperature best agreeing with the measures.

Total radiation and diameters of the stars.—Ratios of total radiation of the stars and sun were determined, and the diameters of perfect radiators of temperature and emission equal to the stars computed. These diameters were compared with interferometer diameters and with Russell's theoretical diameters, and found to be of the same order of magnitude. Better values for these and much fainter stars are expected in 1924.

About seven years ago Director Hale suggested to me the possibility of measuring the energy curves of stellar spectra with the bolometer at the focus of the 100-inch telescope then under construction. Last year, as previously reported,[2] my colleague, Mr. Aldrich, and I, after many discouragements, succeeded in observing the spectra of about ten of the brighter stars in this way. The results were not very accurate, owing to several circumstances enumerated in our paper. It seemed necessary to increase both the accuracy and the sensitiveness of the apparatus in order to get really useful results. Fortunately, Dr. E. F. Nichols, whose

[1] *Contributions from the Mount Wilson Observatory*, No. 280.

[2] Abbot, Fowle, and Aldrich, "The Distribution of Energy in the Spectra of the Sun and Stars," *Smithsonian Miscellaneous Collections*, **74**, No. 7, 1923.

pioneering work in measuring the heat of the stars is well known, offered to have constructed for me a radiometer, which he said would be much more sensitive than the one used at Yerkes Observatory in 1898 and after. As good as his word, Dr. Nichols had constructed by Dr. Tear, of the Nela Research Laboratories, a radiometer with vanes 1.5 mm high, 0.5 mm wide, separated by 2.5 mm. The instrument was stated to have a candle sensitiveness 3.6 times that of the instrument which he used at Yerkes Observatory, and, allowing for the smaller area, 15 times the sensitiveness for equal areas. Considering that the Mount Wilson telescope has 16 times the area of that which Dr. Nichols used, that the sensitiveness factor is fifteen fold, that the altitude of Mount Wilson and the greater clearness of the sky combined should give a factor of nearly twofold, and that it might be possible to increase the scale distance by threefold, there appeared a chance of gaining fully a thousand times in response of the instrument to stellar heat over that which Dr. Nichols perceived. This might well be sufficient to give measurable deflections on the spectra of the brighter stars.

It was not convenient to use the radiometer with the spectroscope which was employed last year for the bolometer, and the new optical arrangement was as follows:

The stellar rays, diverging from the focus *D* (Fig. 1) of the coudé arrangement of the 100-inch telescope, fell at about 16-m distance upon a 50-cm concave mirror *E* (Fig. 2) of 100-cm focus having a central opening of 10 cm. This mirror brought the rays upon a convex mirror *F* situated 20 cm inside the focus. The convex and the concave mirrors had been figured together in the optical shop of the Mount Wilson Observatory so as to reflect a parallel beam through the hole in the 50-cm concave. Behind this mirror the rays fell at minimum deviation upon the face of the 60° flint-glass prism *G* described on page 26 of Volume II of the *Annals of the Astrophysical Observatory of the Smithsonian Institution*. Passing through the prism, the rays fell upon a plane mirror *H* (rotating with the prism and so maintaining minimum deviation) adjusted to send the beam vertically downward to a concave mirror *I* of 45-cm focus, from which they were turned upward to a small plane mirror at 45°, which threw them out of the beam horizontally

upon the vanes of the radiometer *J*. The spectrum (extending horizontally) was allowed to shine through a small window upon one vane only of the radiometer, and the adjustment of the spectrum was examined from behind by means of an eyepiece. Deflections of the radiometer were read on a scale *K*, at 5-m distance, upon which the image of the filament of a hundred-watt Mazda lamp was cast. The scale was drawn with phosphorescent paint on ground glass. Although the mirror on the radiometer had an area less

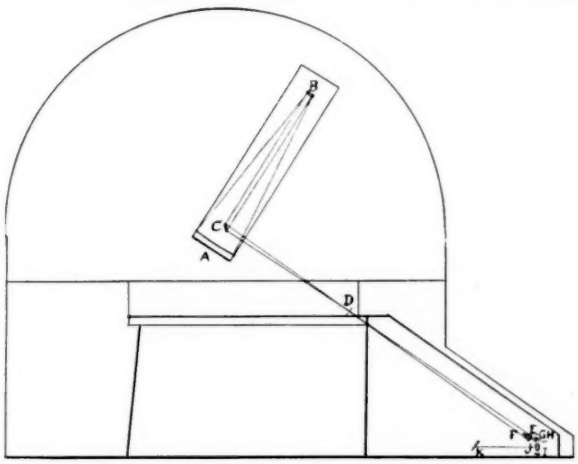

FIG. 1.—Diagram of apparatus. *A*, 100-inch mirror; *B*, convex; *C*, plane; *D*, coudé focus; *E*, 50-cm pierced concave of 1-m focus; *F*, convex; *G*, 60° flint prism; *H*, plane mirror; *I*, concave mirror 45-cm focus; *J*, radiometer; *K*, scale at 5 meters distance.

than 2 sq. mm, the readings were easily made at this long scale-distance. All extraneous light was screened from the radiometer by several diaphragms, appropriately placed and pierced with holes no more than 2 mm in diameter to transmit rays from the Mazda lamp.

The mounting of the prism was provided with a long arm operated by a tangent screw which was turned by a shaft reaching to the position of the observer at the scale of the radiometer. It was designed that each revolution of the screw should move the spectrum along over the radiometer vane through 20 minutes of arc, but certain changes had to be made, so that in the end one revolution shifted the spectrum 21 minutes of arc. The spectrum was oriented

by producing a sodium flame at the coudé focus of the 100-inch telescope so that the *D* line was focused upon the vane of the radiometer. Settings were made in the spectrum at even turns of the screw to the violet and to the red; but in order to avoid the great water-vapor bands in the infra-red, settings were made at $7\frac{1}{2}$ and $9\frac{1}{4}$ turns, so that the whole series of fifteen settings became -6, -5 etc., to the violet of *D*; $+1$, $+2$, etc., $+6$, $+7\frac{1}{2}$, $+9\frac{1}{4}$ to the red. The wave-lengths thus covered extend from 0.437 μ to 2.224 μ. Not all the stars, however, gave measurable deflections over the entire interval.

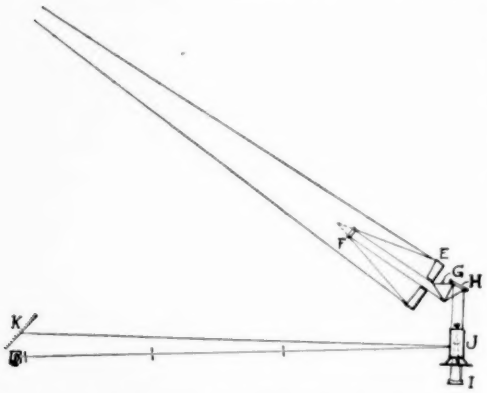

FIG. 2.—Diagram of radiometer and its accessories. *E*, concave mirror; *F*, convex mirror; *G*, 60° flint prism; *H*, plane mirror; *I*, concave mirror, *J*, radiometer; *K*, scale at 5 meters distance.

Observations were made on four nights—October 12, 13, 15, and 18, 1923—with the 100-inch telescope. On the first and last nights, I had the advantage of the aid and advice of Dr. Adams, director of the Observatory, and, on the first three nights, of Messrs. Nicholson and Pettit also, whose long experience with measurements of stellar heat with the thermopile made their aid especially valuable. The night assistant, Mr. W. Klemann, also gave valuable assistance in setting and guiding the telescope.

The procedure was as follows: Having the star image well adjusted at the coudé focus, the assistant kept it central in the small aperture of a screen placed at that point, which was at first about 2 mm in diameter but afterward was enlarged somewhat. He perceived the image as it fell upon the white back of the shutter inside the screen, operated by a cord from the radiometer scale. Thus, when the shutter was closed, the assistant would assure himself that the star was accurately central, so that when the shutter was opened it was quite certain that the beam would fall properly

upon the radiometer vane. The program of observing was first to set the star spectrum as far to the violet as measurable deflections were obtained; and from there, at an even number of turns from the *D* line, to begin observing. Usually the shutter was opened 15 seconds, closed 15 seconds, opened 15 seconds, and so on, until eight readings had been made. Then the spectrum was moved to the next position, the process repeated, and so on, as far down in the infra-red as it was found worth while to go. Then a duplicate series of measurements was made, beginning with the infra-red and ending at the original position in the violet.

Thus, in most cases, the intensities found at the several spectral points depend on sixteen readings of the radiometer. In some cases, however, if the deflections were large, as with the sun, with Betelgeuse, and with Sirius, it seemed unnecessary to multiply them so extensively.

Of the four nights available, the first and last were favored with good seeing, but on the second and third a strong north wind made the seeing too bad for satisfactory observations. Moreover, on the first night the instruments did not work very satisfactorily, so that the readings for that night are hardly of sufficient weight to make it worth while including them with those of the last night when conditions were almost entirely satisfactory. Unfortunately, a thin cirrus partly covered the sky during the early part of the last night, October 18, 1923, so that the observations of α Herculis, Vega, β Pegasi, and Capella are not quite so successful as those made during the last half of that night, when not only the seeing but the transparency was fine. The writer made all the observations of deflections upon the radiometer scale, and these were recorded by Messrs. Adams, Pettit, or Nicholson.

COMPARISON WITH THE SUN

From many years of solar constant work on Mount Wilson, the dispersion of the prism used, the form of the sun's energy curve, the intensity of its radiation, and the transmission by our atmosphere for different wave-lengths are well known. By taking advantage of these data, the stellar measures could readily be reduced to standard conditions. This, however, required that the solar spectrum

should be observed with the 100-inch telescope and radiometer, following as nearly as possible the procedure for other stars. For this purpose, a screen was placed on the end of the telescope tube, having eight $\frac{1}{4}$-inch holes in thin brass plates symmetrically placed on a circle of about 5 feet diameter. At the coudé focus, another screen was fixed, having a thin brass plate pierced with a Number 60 twist drill hole.[1]

The approximate reduction of the solar beam as compared with the stellar beams was[2]

$$18{,}100 \times 491{,}000 = 0.89 \times 10^{10} .$$

It is a pity that this value was not determined with higher accuracy, for it affects some computations of stellar diameters presented at the end of the paper. Any change in this factor will produce a change in the calculated stellar diameters varying inversely as the square root of the factor.

One other factor is required. This depends on the fact that the solar constant, 1.94 calories per sq. cm per minute, represents the mixture of all solar rays, whereas the pin-hole made by the Number 60 drill was situated in the center of the 28-inch solar image at the coudé focus. According to Abbot's *The Sun*, page 110, the average solar ray is fainter than the central solar ray in the ratio $1.407/1.620 = 0.87$. Proper allowance for this is made wherever needed below.

A partial check on the accuracy of the adopted value, $0.89 \cdot 10^{10}$, is furnished by comparing my results on the ratios which I have called N, Table VII, of total star heat and sun heat outside our atmosphere, with results which Dr. Pettit was kind enough to communicate in advance of publication, giving the determinations by Pettit and Nicholson of stellar radiation in the zenith at Mount Wilson, expressed in calories per sq. cm per minute. This comparison requires some assumption as to the transmission of the atmosphere for vertical star rays. For Capella, I assume that the value 81.7 per cent given for solar rays on page 296 of my book *The Sun* is applicable. But for the much hotter stars, the absorp-

[1] The measured diameter of the hole is 1.029 mm.

[2] Foreshortening of the small temporary coudé mirror required allowances of 30 and 20 per cent for Capella and Vega, respectively.

tion of the air for ultra-violet rays, especially beyond 0.3 μ, and for the giant K- and M-type stars, the absorption by atmospheric humidity and carbon dioxide on the extension of the spectrum to long-wave infra-red rays must for each class lead to smaller values. Rough estimates appear in the second line of Table I, which,

TABLE I

COMPARISON OF ABBOT'S AND PETTIT'S VALUES

	Sirius	Procyon	Capella	Aldebaran	Betelgeuse
Atmospheric transmission $=A$..	0.60	0.70	0.82	0.73	0.64
$N\times 10^{11}$	6.6	1.2	2.2	2.5	7.9
$1.94\,NA\times 10^{12}$	76	17	34	36	98
Pettit $\times 10^{12}$	58.1	13.7	25.1	29.1	77.3
Abbot−Pettit, per cent	+24	+20	+26	+20	+21

combined with the values of N from Table VII and with the solar constant, 1.94 calories, may be compared with Pettit's figures in the fifth line. Considering the several doubtful factors involved, the agreement is quite as good as could be expected, but may indicate a real error of 20 per cent. If so, my star diameters given in Table III should each be decreased by 11 per cent.

THE OBSERVATIONS

As a fair illustration of the character and agreement of the observations, the measures for Aldebaran are given in full in Table II. For the remaining stars, including the sun, only mean values are given (Table III).

The headings of Table II give the position in the spectrum and the Pacific Standard Time of observation. The first column of each division gives the readings with the shutter alternately open and closed; the second, the means of alternate readings in the first column. The differences, in the third column, are the deflections. Values in parentheses were omitted in forming the means. This method of reduction, in effect, gives different weights to the separate observations. It was adopted as one permitting ready scrutiny of separate values, so that wild ones might be rejected, which occurs, in fact, above. As will easily be seen by taking differences, the main part of the error in almost all cases comes from wandering of the light-spot on the reading scale, not from bad following or

TABLE II

COMPLETE OBSERVATIONS OF ALDEBARAN ON OCTOBER 18, 1923

D, 13^h11^m			+1, 13^h13^m			+2, 13^h15^m		
74	.....	.....	70	.....	.....	73	.....	.....
72	71.5	0.5	74	69.5	4.5	71	70.5	0.5
69	73.5	4.5	69	73.5	4.5	68	73.0	5.0
75	72.5	2.5	73	69.5	3.5	75	70.5	4.5
76	75.0	−1.0	70	72.5	2.5	73	73.5	0.5
75	76.5	1.5	72	70.0	2.0	72	72.0	0.0
77	73.5	−3.5	70	74.0	4.0	71	72.5	1.5
72	.....	.....	76	.....	.....	73	.....	.....
Means ...	.....	0.3	.....	.....	3.5	.....	.....	2.0

+3, 12^h17^m			+4, 13^h20^m			+5, 13^h23^m		
68	.....	.....	61	.....	.....	58	.....	.....
73	67.5	5.5	75	59.5	15.5	73	57.0	16.0
67	74.0	7.0	58	74.0	16.0	56	70.5	14.5
75	68.0	7.0	73	59.5	13.5	68	55.0	13.0
69	74.5	5.5	61	75.0	14.0	54	70.0	16.0
74	69.0	5.0	77	60.5	16.5	72	55.0	17.0
69	75.0	6.0	60	74.5	14.5	56	72.0	16.0
76	.....	.....	72	.....	.....	72	.....	.....
Means...	.....	6.0	.....	.....	15.0	.....	.....	15.4

+6, 13^h26^m			+7.5, 13^h26^m			+9½, 13^h31^m		
56	.....	.....	55	.....	.....	68	.....	.....
72	55.0	17.0	70	55.5	14.5	72	68.5	3.5
54	70.0	16.0	56	69.5	13.5	69	72.5	3.5
68	56.0	12.0	69	56.0	13.0	73	69.5	3.5
58	71.5	13.5	56	69.5	13.5	70	72.5	2.5
75	55.5	19.5	70	56.0	14.0	72	69.5	2.5
53	72.0	19.0	56	70.5	14.5	69	72.5	3.5
69	.....	.....	71	.....	.....	73	.....	.....
Means ...	.....	16.2	.....	.....	13.8	.....	.....	3.2

+9½, 13^h35^m			+7.5, 13^h28^m			+6, 13^h41^m		
68	.....	.....	59	.....	.....	57	.....	.....
72	69.5	2.5	72	58.5	13.5	69	54.5	14.5
69	72.5	3.5	58	71.5	13.5	52	69.0	17.0
73	69.0	4.0	71	57.5	13.5	69	54.0	15.0
69	73.0	4.0	57	69.5	12.5	56	70.5	14.5
73	68.0	5.0	68	56.5	11.5	72	54.0	18.0
67	74.0	7.0	56	68.5	12.5	52	70.0	18.0
75	.....	.....	69	.....	.....	68	.....	.....
Means ...	.....	4.3	.....	.....	12.8	.....	.....	16.2

TABLE II—*Continued*

+5, 13^h44^m			+4, 12^h47^m			+3, 13^h48^m		
57	.....	.....	63	.....	.....	65	.....	.....
69	56.5	12.5	70	61.5	8.5	72	63.5	8.5
56	70.5	14.5	60	70.0	10.0	62	70.5	8.5
71	57.0	14.0	70	58.0	12.0	69	66.0	(3.0)
58	71.0	13.0	56	68.5	12.5	70	70.0	(0.0)
71	57.5	13.5	67	58.5	8.5	71	66.0	5.0
57	70.0	13.0	61	69.5	8.5	62	72.5	10.5
69	.....	.....	72	.....	.....	74	.....	.....
Means ...	.....	13.4	.....	.....	10.0	.....	.....	8.1

+2, 13^h51^m			+1, 13^h53^m			*D*, 13^h56^m		
68	.....	.....	68	.....	.....	70	.....	.....
72	67.0	5.0	73	69.0	4.0	71	70.5	0.5
66	71.5	5.5	70	70.0	0.0	71	72.0	1.0
71	64.5	6.5	67	70.0	−3.0	73	72.5	0.5
63	71.5	8.5	70	69.5	−0.5	74	72.5	−1.5
72	65.5	6.5	72	70.5	1.5	72	72.5	−0.5
68	72.5	4.5	71	72.5	1.5	71	72.0	1.0
73	.....	.....	73	.....	.....	72	.....	.....
Means ...	.....	6.1	.....	.....	0.6	.....	.....	0.2

bad sky. Hence, as happens in this method, it is rather justifiable to give first and last readings little weight.

The results in Table III, on the prismatic scale, are illustrated in Figure 3. The double maximum for Rigel, the difference of

TABLE III

MEAN RADIOMETER DEFLECTIONS IN MILLIMETERS

PLACE		SUN	β ORIONIS	α LYRAE	α CAN. MAJ.	α CAN. MIN.	α AURIGAE	α TAURI	β PEGASI	α ORIONIS	α HERCULIS
Prismatic	Wave-Length										
−6	0.437	3.3	.....	.....	.....	.....	.....	.....	.....	.....	.....
−5	.454	5.6	.....	2.0	.....	.....	.....	.....	.....	.....	.....
−4	.472	4.8	−0.6	2.1	.....	.....	.....	.....	.....	.....	.....
−3	.494	10.8	+1.7	4.2	2.2	.....	0.1	.....	.....	.....	.....
−2	.520	16.4	1.4	4.2	5.1	.....	2.0	.....	.....	.....	.....
−1	.551	19.9	3.8	5.2	7.5	−1.1	0.8	.....	.....	1.5	.....
D	.589	23.9	1.2	3.1	7.5	0.6	2.4	0.3	0.4	2.1	− 0.1
+1	.636	33.0	0.8	3.6	10.5	2.0	3.5	2.1	0.8	3.5	1.5
+2	.700	39.5	1.3	1.8	11.4	3.8	3.1	4.0	1.5	6.4	2.8
+3	.784	46.4	4.5	0.4	11.8	4.0	5.2	7.0	2.8	18.7	3.8
+4	0.905	53.6	3.0	.....	10.6	3.8	7.4	12.5	5.1	27.8	7.3
+5	1.080	55.7	+3.2	.....	9.3	4.6	9.0	14.4	5.3	43.2	19.5
+6	1.316	42.0	−0.2	.....	7.1	3.0	6.1	16.2	6.8	48.4	21.1
+7½	1.751	25.9	.....	.....	3.9	4.8	3.0	13.3	6.4	44.0	23.3
+9½	2.224	6.2	.....	.....	1.4	−1.5	0.6	3.8	2.9	16.4	−5.5
Range in air-mass	.....	1.59	1.42	1.35	1.59	1.30	1.68	1.10	1.01	1.14	1.73
	.....	1.74	1.35	1.64	1.58	1.22	1.35	1.06	1.04	1.12	2.20

place of maximum for Vega and Sirius, both said to be type Ao, and the peculiar distribution for Procyon, are all very notable features of these prismatic energy curves. It is a pity that Rigel, Sirius, Procyon, Capella, and α Herculis are all double stars, for the companions may in some cases complicate the curves. One longs for more data of the several types so as to determine whether these

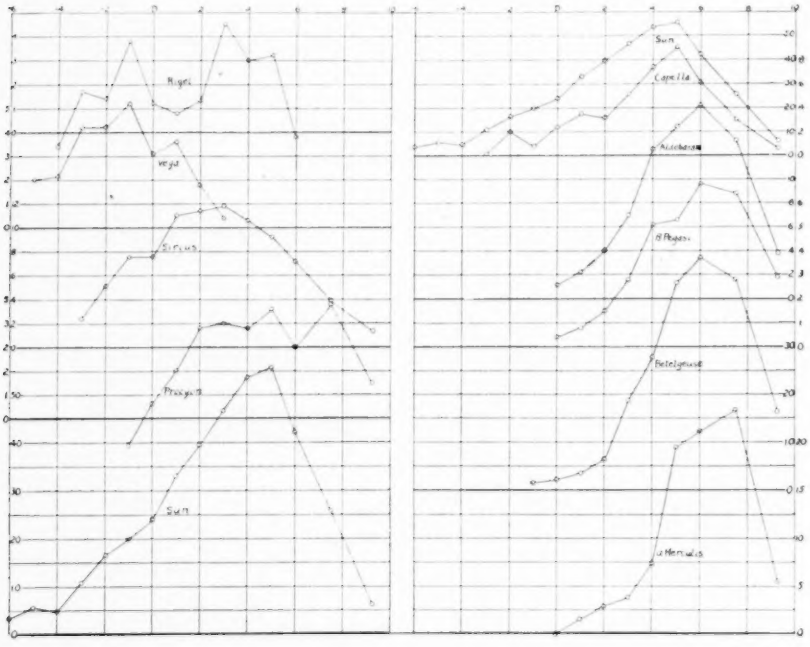

FIG. 3.—Prismatic energy of spectra

peculiar features are characteristic of the types or merely of the individual stars. It will be recalled that a double maximum for Rigel was found by Abbot and Aldrich in the bolometric work of 1922.

REDUCTIONS OF OBSERVATIONS

As indicated in Table II, the Pacific Standard Time of each set of measurements was recorded. From this, the hour angles of the stars were readily found. Tables of air-masses at Mount Wilson are available for all declinations and hour angles suitable to the

solar-constant observations which used to be made there. These tables covered nearly all cases, and with a few supplementary computations gave the air-masses of observation, as shown in Table III. With the known length of the tangent arm and the pitch of the screw, the positions in the spectrum were all readily reduced to the form of prismatic deviations from *D*. Then by comparison with Table XVI, page 105, Volume II, *Annals of the Astrophysical Observatory of the Smithsonian Institution*, these positions were reduced to wave-lengths as given in the second column of Table III above.

Atmospheric transmission coefficients for these wave-lengths were assumed to conform to the mean values given in Table XVII, *ibid.*, Volume II, page 111. As the mean air-mass prevailing during the observations, considered as a whole, was near 1.30, it was decided to reduce all data to this air-mass by employing the atmospheric transmission coefficients just named. Since the differences of air-mass from 1.30 seldom exceeded 0.3 air-mass (except for α Herculis, where they reached 0.62), the corrections were small, seldom reaching 5 per cent. In this way, all the data were placed on a comparable footing, excepting, of course, for the enormous instrumental reduction factor required by the solar deflections. But as this factor is constant for all wave-lengths, it does not affect the further reductions to normal scale outside our atmosphere.

In order to make this further reduction, the solar observations with the 100-inch telescope, reduced to air-mass 1.3, were compared with the solar-energy distribution outside the atmosphere as determined in recent years at Mount Wilson.[1] By dividing the 100-inch observations into the normal values outside the atmosphere, factors resulted, which, when smoothed, are applicable to reduce all the stellar values for air-mass 1.3 to normal values outside the atmosphere. These data appear in Table IV.

Applying these correcting factors to the stellar data as reduced to air-mass 1.3, we arrive at the normal energy distributions outside the atmosphere shown in Table V, all observed with the 100-inch, but with reductions based on the normal solar distribution determined by Messrs. Abbot, Fowle, and Aldrich in 1920 and 1922. These results are given graphically in Figure 5 (p. 102).

[1] See *Smithsonian Miscellaneous Collections*, **74**, No. 7, 1923.

TABLE IV

SOLAR DATA

PLACE		ATMOSPHERIC TRANSMISSION	AIR-MASS *minus* 1.3	PERCENTAGE CORRECTION	SOLAR ENERGY CORRECTED	NORMAL OUTSIDE ATMOSPHERE	FACTOR	FACTOR, SMOOTHED
Prismatic	Wave-Length							
−6	0.437	0.788	0.24	5.6	35	490	1400	1400
−5	.454	.812	.25	5.1	59	550	933	910
−4	.472	.829	.26	4.7	50	565	1130	620
−3	.494	.843	.27	4.5	113	552	489	460
−2	.520	.863	.28	4.1	171	520	304	335
−1	.551	.874	.29	3.9	207	500	242	253
D	.589	.878	.30	3.9	248	485	196	188
+1	.636	.909	.31	3.0	340	445	131	137
+2	.700	.936	.32	2.2	404	392	97.0	96.0
+3	.784	.957	.33	1.5	471	302	64.1	65.0
+4	0.905	.965	.34	1.3	543	220	40.5	41.0
+5	1.080	.969	.35	1.2	564	165	29.3	30.0
+6	1.316	.973	.36	1.0	424	108	25.5	25.0
+7½	1.751	.973	.37	1.1	262	55	21.0	21.0
+9¼	2.224	0.965	0.39	1.3	63	16	25.4	25.0

TABLE V

STELLAR ENERGY CURVES OUTSIDE ATMOSPHERE ON NORMAL SCALE*

Wave-Length	Sun	β Orionis	α Lyrae	α Can. Maj.	α Can. Min.	α Aurigae	α Tauri	β Pegasi	α Orionis	α Herculis
0.437	490	.....	.....	.....	.....	.....	.....	.....	.....	.....
.454	537	.....	1910	.....	.....	.....	.....	.....	.....	.....
.472	310	∞(?)	1432	.....	.....	.....	.....	.....	.....	.....
.494	510	782	1988	1058	.....	46	.....	.....	.....	.....
.520	573	469	1440	1776	.....	703	.....	.....	.....	.....
.551	524	961	1341	1948	∞(?)	202	.....	.....	379	.....
.589	466	226	602	1448	113	470	56	75	378	∞(?)
.636	462	110	517	1466	274	480	276	110	466	219
.700	388	125	173	1114	365	298	374	144	614	278
.784	306	292	36	773	260	338	448	182	1202	253
0.905	223	123	.....	443	156	303	508	205	1132	307
1.080	169	96	.....	282	138	270	432	156	1287	597
1.316	106	∞(?)	.....	177	75	152	405	167	1202	537
1.751	55	.....	.....	82	101	63	277	132	918	498
2.224	16	.....	.....	35	38	15	95	72	407	140

* These results are given graphically in Fig. 5.

STAR TEMPERATURES AND DIAMETERS

Perhaps a wiser man would end here, and avoid incumbering the literature with values which we may expect will be superseded by better ones within another year. But the temptation is too strong, and I must ask indulgence while venturing to combine these results with the data of the perfect radiator.

The radiometer observations cover pretty well the extent of spectrum occupied by the radiation of K and M stars, but, owing to the high prismatic dispersion of blue and violet rays, they are too inaccurate in these spectral regions to be serviceable for the stars of classes B–G. Hence, I have attempted to supplement the radiometric work by the spectral-photometric results of Wilsing, Scheiner, Münch, Rosenberg, and Plaskett. These observers, one or more of them, have examined the spectra of all ten stars excepting α Herculis, for which, indeed, short-wave energy spectra are less important to the purpose.

TABLE VI

WILSING'S DATA

STAR	0.448	0.480	0.513	0.584	0.638
	Five Wave-Lengths Series				
α Lyrae	+0.136	+0.100	+0.068	−0.118	−0.185
α Can. Min	+0.027	+0.030	+0.064	−0.003	−0.116
α Tauri	−0.306	−0.124	−0.049	+0.245	+0.236
β Pegasi	−0.396	−0.241	−0.009	+0.239	+0.391
	Ten Wave-Lengths Series				
	0.451	0.472	0.494	0.514	0.535
α Lyrae	+0.119	+0.085	+0.066	+0.070	+0.059
α Can. Min	.035	.118	.143	.063	−0.027
α Aurigae	+0.037	+0.069	+0.192	+0.142	+0.006
α Tauri	−0.143	−0.056	−0.014	−0.110	−0.015
	0.556	0.577	0.593	0.615	0.642
α Lyrae	+0.039	−0.052	−0.095	−0.126	−0.165
α Can. Min	+0.013	.042	.055	.129	.113
α Aurigae	−0.089	−0.108	−0.003	−0.162	−0.082
α Tauri	+0.040	+0.068	+0.054	+0.077	+0.100

Different steps were required to piece the work of these observers on to mine. Wilsing[1] has collected and revised the visual spectral-photometric work of Wilsing, Scheiner, and Münch. He gives the logarithmic values of relative spectral intensities in two series, for five and ten wave-lengths, respectively, as listed in Table VI.

[1] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 74, 1919.

For the present purpose it was only necessary to interpolate from the radiometric curves for these stars the values corresponding to Wilsing's wave-lengths, take corresponding logarithms, and thereby determine the best logarithmic factor to reduce his scale to mine. This done, Wilsing's values were converted to natural numbers and entered in the plots, Figure 5.

As for the photographic spectral-photometric investigation of Rosenberg,[1] his results are expressed primarily with reference to the star α Aquilae, and secondarily to a hypothetical star representative of Miss Maury's type II. Rosenberg observed the sun's spectrum, and this forms a connecting link to my investigation. But Wilsing[2] has discussed Rosenberg's work on the solar spectrum at great length, and disapproves of his application of a correction for loss by diffraction. Brill[3] agrees with Wilsing in this view, which I venture to adopt, and hence employ Rosenberg's solar energy spectrum without the diffraction corrections. Brill, indeed, reduces Rosenberg's results to actual intensities, but his process is somewhat involved, and I have preferred to go about it independently as follows:

I take from Brill's "Tabelle X," of the article just cited, corrected values of Rosenberg's spectral mean differences in stellar magnitudes for sun *minus* α Aquilae. From Rosenberg's memoir just cited, "Tabelle V," I take the corresponding values of α Aquilae *minus* Maury type II. Adding these quantities algebraically, and multiplying by 0.4, I reduce to common logarithmic form the intensity-curve for sun *minus* Maury type II. Then for corresponding wave-lengths I take the logarithmic values of the intensity of the solar spectrum from Abbot, Fowle, and Aldrich, above cited, add them to what has just been described, and thus obtain the logarithmic spectral intensities for Brill's hypothetical star of Maury's type II. The curve is not very smooth, as appears in Figure 4, and considering how difficult the long-wave end of it must have been for Rosenberg who observed photographically, and how difficult

[1] *Abhandlungen der Kaiser Leopold-Carolinischen Deutschen Akademie der Naturforscher*, **101**, 1915.

[2] *Astronomische Nachrichten*, **204**, 153 (No. 4881), 1917.

[3] *Ibid.*, **218** (Nos. 5222–5223), 1923.

the short-wave end of it is for both him and everybody else on account of atmospheric and other difficulties, I have thought best to smooth the curve as shown in Figure 4. This smoothing is rather summary, and results in intensity differences from the direct results reaching 20 per cent in the worst cases; but this, as will appear, is not very serious for a preliminary study as rough as the present one.

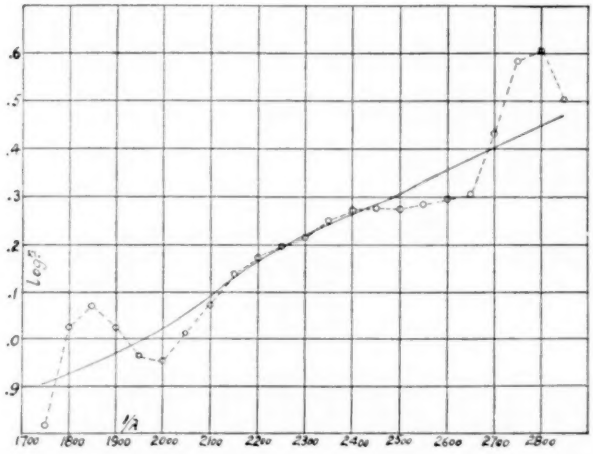

FIG. 4.—Rosenberg-Abbot logarithmic intensities for Maury type II

The way remaining is easy: Read from Rosenberg's plotted data, Plates X–XXIX of his publication, the mean difference in stellar magnitude, star *minus* Maury type II, multiply by 0.4, and subtract the result from the smoothed logarithms for Maury type II, and then take logarithms from the radiometer curves for corresponding wave-lengths. The logarithmic factor thus obtained reduces Rosenberg to the scale of the radiometer. The corresponding natural numbers are plotted in Figure 5. Treated in this way Rosenberg's data cover all the stars except β Pegasi and α Herculis.

From Plaskett's work[1] come only two stars, the sun and α Aurigae. His values are given, in Tables XIV and XVI of his publication, directly as intensities, and had merely to be reduced

[1] *Publications of the Dominion Astrophysical Observatory*, **3**, No. 12, 1923.

to the radiometric scale. I have used his data only in preliminary reductions. They are not plotted in Figure 5.

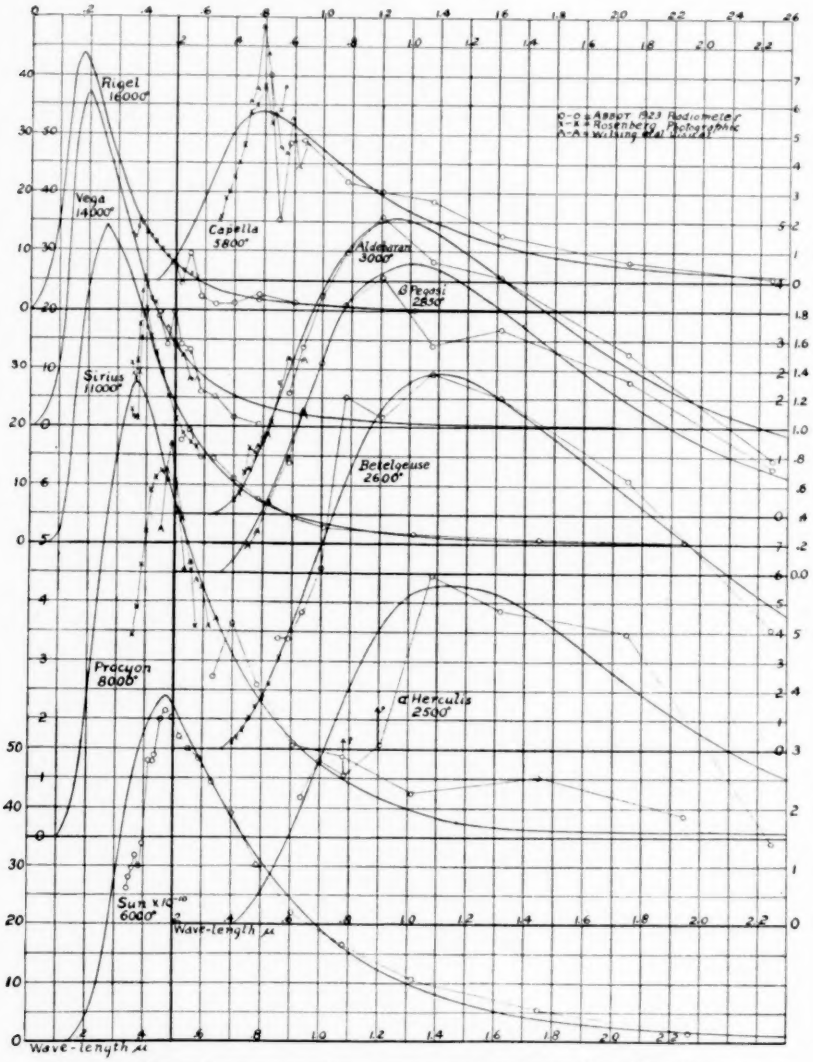

FIG. 5.—Normal energy-spectrum curves and corresponding black-body curves. Various scales of ordinates; two scales of abscissae.

It will be seen from Figure 5 that generally the agreement is good in respect to the march of spectral energy curves determined

by the several observing sources. The results obtained with radiometer, however, are nearly valueless for wave-lengths less than 0.55 μ, and the Rosenberg-Abbot data[1] become so beyond wave-length 0.38 μ. On the whole, the addition of the spectral-photometric results, visual and photographic, has greatly improved the curves for purposes of comparison with black-body data, which is now to be attempted. Black-body curves best fitting the data were drawn for each star, as shown in Figure 5. Numerous trials were made before settling upon what seemed the most suitable temperature. The roughly equivalent black-body temperatures of the ten stars according to this criterion are as given in Table VII.

The agreement between the observations and the temperature curves is fairly good. The exceptions are Rigel, Vega, Procyon, Capella, and α Herculis. In the case of Rigel, the double maximum, which appeared on the prismatic curve so strongly, now exhibits itself as an absorption band in red. Vega shows a depression in the same region but does not recover in the infra-red. Probably, though, this defect in Vega is due to error and attributable to the streaky sky at the time of the observations. For Capella and α Herculis also, the sky conditions were poor during the radiometric observations, which very likely accounts for some of their irregularities. The curve for α Herculis shows two possible changes corresponding to the different weighting of the observations. The observations of Capella by Wilsing and Rosenberg are also irregular. As for Procyon, the large positive departure in the infra-red may possibly be the effect of the companion, but this is merely a suggestion.

It is interesting, now, to determine the diameters of spherical black bodies which would send to the earth as much radiation as the observed stars do. The theory is simple:

Let R = radius of equivalent black body in miles
T = absolute temperature
D = distance of the star in miles
N = fraction stellar radiation bears to solar radiation outside our atmosphere

[1] I ought, perhaps, to apologize to Rosenberg for treating his work so freely.

Other symbols: Π, parallax; π, geometrical factor, σ, radiation constant; and 1.94, solar constant. Then

$$\frac{4\pi R^2 \sigma T^4}{4\Pi D^2} = 1.94\, N\,,$$

or, approximately,

$$R = D\sqrt{\frac{1.94\, N}{8\times 10^{-11}\, T^4}}\,,$$

where

$$D = \frac{93\times 10^6}{5\times 10^{-6}\times \Pi}\,.$$

T is the black-body temperature corresponding to the Wien-Planck curve which best represents the stellar observations. N is determined by comparing the area under the stellar curve with that under the solar curve, both taken with the radiometer and 100-inch telescope, corrected for atmospheric absorption, and pieced out in the region of short wave-lengths by the spectral-photometric and black-body curves. The area of the solar curve has been reduced by the ratio 1.407/1.620, as stated above.

For the hot stars of types B–F, there is much uncertainty as to the correction for atmospheric absorption, because so large a part of the area of the corresponding black-body curve lies among wave-lengths for which the atmosphere prevents observation. Assuming that the numerous ultra-violet ultra-ionized metallic lines will produce great depletion of these short wave-lengths within the star itself, I have adopted the convention of using in the case of the stars only half of the black-body areas for wave-length less than 0.3 μ. The results found in this way are in Table VII.

I am much indebted to Dr. Adams for values of parallaxes, and, through him, to Mr. Pease for latest information on interferometer measures of star diameters, given in the sixth column of the table. The value given for Sirius is stated to be "merely an estimate from the visibility fringes and not of much weight." The values headed "Russell" are from a letter kindly sent me by Dr. Russell, giving

angular diameters in seconds of arc, computed after the manner he developed in *Publications of the Astronomical Society of the Pacific*.[1] In this letter he states that the new values are found from the old by adding about 22 per cent "to make them fit the Mount Wilson measures for Arcturus, Antares, Betelgeuse, and Aldebaran."

TABLE VII

STELLAR TEMPERATURES, RADIATION AND DIAMETERS

STAR	ABSOLUTE TEMPERATURE C.	N^* UNIT $=10^{-11}$	PARALLAX	SUN'S DIAMETER $=1$†		
				Radiometer	Interferometer	Russell
Sun	6,000°	.....	.....	.....	.....	.....
β Orionis	16,000	3.20	0".007	20.	.....	28.
α Lyrae	14,000	6.10	0.130	2.0	.....	3.0
α Can. Maj.	11,000	6.60	0.370	1.2	2.0	.....
α Can. Min.	8,000	1.24	0.315	1.1	.....	1.6
α Aurigae	5,800	2.20	0.071	13.	.....	9.
α Tauri	3,000	2.54	0.053	70.	39.	.....
β Pegasi	2,850	1.10	0.026	94.	82.	.....
β Orionis	2,600	7.90	0.017	510.	280.	.....
α Herculis	2,500	3.60	0.007 0.013	900. 480.	.....	230.

* Ratio of stellar to solar radiation outside earth's atmosphere.

† To express in kilometers, multiply by 1.42×10^6; to express in miles, multiply by 0.865×10^6.

The parallax of α Herculis is that found by Adams spectroscopically for the red star. He found 0".018 for the G8-type companion. Burnham's double-star catalogue, while it does not indicate any revolution of the two, clearly shows them to have sensibly the same proper motion. If this should be taken as indicating a true binary character, and hence that the two stars are at the same distance from the earth, a parallax of 0".013 might be preferred. My figures for diameter would then be reduced correspondingly, but this would similarly reduce Russell's diameter.

It is very noticeable that my diameters fall below the interferometer and other values for hot stars, agree with them for solar type, and exceed them for the giant K and M stars. If the factor of reduction from solar to stellar radiometry, taken as $0.89\cdot10^{10}$,

[1] *Op. cit.*, **32**, 307–317, 1920.

should prove different, it would make the disagreement larger at one end of the spectral sequence and reduce it at the other. If the error lies in underestimating areas of curves in the ultra-violet, it must be supposed that I have cut off far too much there. It is, indeed, out of the question to add enough area to bring accord in this way. As for overestimating the areas of curves for the giant stars, here again, although I have included all the area under the black-body curves extended to infinitely great wave-lengths, the omission of it would not suffice to bring accord.

The difference must lie in the temperatures chosen. As I have pointed out elsewhere, the sun's energy curve fits no black-body curve whatever. It is too high in the infra-red and too low in the violet when fitted to the 6000° curve between 0.5 and 1.0 μ. But if fitted in these regions to a black-body curve of low temperature, the fit is lost in the best-determined part, 0.5–1.0 μ. This circumstance doubtless comes about because we see much deeper into the sun at longer wave-lengths and hence see a hotter sun for long-wave rays than for shorter ones.

May not this point to the main cause of discrepancy? For the giant K and M stars, the long wave-lengths may come from deep within the star, while the visible wave-lengths originate at lesser depths. Consequently, as the infra-red is relatively too high, in trying to fit the form of curve we take a temperature which is really too low to suit the actual average temperature of emission, and the result is too large a diameter. For the hot stars, this tendency does not prevail, because their infra-red intensities are so small that they necessarily receive very small weight in the selection of the proper temperature curve. That they give diameters which are too small, if indeed this should prove to be the case when greater accuracy is attained in this and other methods of investigating diameters, may arise partly from an error in the solar reduction constant $0.89 \cdot 10^{10}$ mentioned above. On the other hand, these stars may lack much of being full radiators, so that the black-body constant σ may be too large for them.

At all events, the substantial variation between diameters computed from these new observations, while wider in its limits than that estimated by other observers in other ways, is yet so

nearly accordant with their results in its range as to confirm, if confirmation is needed, the prevailing views of stellar constitution. Through more accurate radiometric observations, undoubtedly to be obtained in the future, we may hope to throw light on the discrepancies which the forms of the curves reveal between stars of similar type and between stellar and black-body energy curves.

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SOME WAVE-LENGTHS IN THE VACUUM-ARC SPECTRUM OF TITANIUM

By HENRY CREW

ABSTRACT

Wave-length measurements in the vacuum-arc spectrum of titanium, from λ 3653 to λ 6366.—Measurements have been made of 700 known lines, and of 78 new lines, apparently belonging to titanium.

The temperature control, using a new form of toluene thermostat, is believed to keep the temperature of the grating constant to within 0°.01 C. Wave-lengths are given to 0.001 A. A discussion of the relative merits of the Rowland and Littrow mountings is included.

The spectral lines of titanium are of interest partly because they belong to one of the chemical elements, partly because of their frequent occurrence in stellar spectra, partly because they are often used, as suggested by Professor Frost, for comparison lines in the measurement of stellar spectra, and partly because of certain regularities[1] which have recently been discovered among these lines. In view of the fact that no one has yet determined the wave-lengths of titanium in the vacuum-arc, and, since there are numerous discrepancies between the best two or three determinations which already exist, it seemed worth while to make the following measurements.

SOURCE

The source of the spectrum here measured was a 220-volt D.C. arc, using electrodes of metallic titanium, held in iron clamps. Cast titanium was broken up into pieces of size and shape convenient for use as electrodes. Portions of the spectrum photographed were made by using as electrodes cylinders of titanium which the Fansteel Products Company were good enough to work into this shape in a resistance furnace filled with hydrogen.

This titanium arc was worked in a vacuum chamber, designed by Dr. George V. McCauley and myself and fully described by Professor F. L. Brown in this *Journal* [**56,** 56, 1922]. The pressure in the vacuum chamber ranged from 5 to 15 mm of mercury.

[1] Kiess, C. C., and Harriet K., *Jour. Wash. Acad. Sci.*, **13,** 270, 1923.

The other conditions under which the arc was worked[1] were as nearly as possible those recommended by the Solar Union in 1913 for the determination of tertiary standards, namely, length of arc, 6 mm; current, from 4 to 6 amp.; slit at right angles to the arc so as to use only the middle portion of the arc. Since the arc and the slit of the spectrograph were each vertical, it was necessary to insert, between the arc and its image on the slit, a rotating prism with refracting angle of 90°.

The comparison spectrum was that of the standard Pfund iron-arc of 220 volts D.C., using 4 to 6 amp., with the image projected at right angles to the slit, as in the case of titanium. Some of the spectrograms were made by the half-exposure method, first giving the iron half its proper exposure, then giving the titanium its full exposure, then finally giving the iron the second half of its exposure. But most of the photographs were made by simultaneous exposure. To accomplish this a concave mirror is used to project an achromatic image of the iron-arc upon the region between the titanium electrodes. When the beam reflected from the concave mirror is properly collimated with the axis of the image lens, both arcs are in focus on the slit at the same time.

On one side of the spectrum to be measured was photographed the spectrum of titanium alone; on the other side, the spectrum of iron alone. Each of these side spectra was given a somewhat longer exposure time than the middle strip, so that in identifying lines one might feel certain that the apparent absence of a line in the side spectrum was not owing to underexposure.

THE SPECTROGRAPH

The dispersion piece of the instrument is a 6-inch plane grating, ruled by Professor Michelson and provided with a Littrow mounting. The collimating lens has a clear aperture of 6 inches. It was made by Brashear and designed by Hastings for a minimum focal length of 25 feet at λ 4530. The grating is one of fine definition and, being ruled with approximately 17,000 to the inch, has in the second order (where it is mostly used) a resolving power of 200,000, and a dispersion of almost exactly 1 angstrom to a millimeter. The ghosts

[1] *Astrophysical Journal*, **39**, 93, 1914.

in this grating are rather strong and have to be carefully watched when it comes to the identification of lines. The slit of the spectrograph is placed 12 cm above the level of the photographic plate, and is located 14 cm nearer the grating than is the photographic plate. The collimating lens, the grating, and their housing are all situated on one large brick pier. The slit, the image lens, the rotating prism, and the camera are all placed on another large pier. There is no connection between the two piers or between either pier and the building, except, of course, through the ground.

When an absorbent was needed to separate overlying spectra, an ammoniacal solution of aesculin was used. The photographic plates employed in the red were Wratten and Wainright's Panchromatic; some of them were bathed in ammonia just before use in order to increase their sensibility. In the shorter wave-lengths, Cramer Crown and Seed Gilt Edge plates were used.

THE THERMOSTAT

It is well known that a spectrograph of this type is extremely sensitive to changes of temperature. Some careful measurements on this instrument made several years ago by Professor F. L. Brown show that the photographic image on the plate shifts by 0.001 angstrom for a variation of temperature amounting to only 0°.01 C. At Mount Wilson, uniformity of temperature is secured by placing the grating at the bottom of a pit. Here we have tried to accomplish the same end by use of a thermostat. The grating is completely inclosed in a double-walled case made of sheet-zinc. The inner wall is uniformly and closely wound with No. 27 enameled wire, which serves as a heating coil. The space between the two walls is filled with felt. The temperature is held constant—generally at 2 or 3 degrees above the highest room-temperature to be expected—by placing inside the case and just behind the grating a glass bulb containing approximately 100 cc of toluene. At the top of this bulb is attached the usual U-tube with a mercury contact. As soon as the proper temperature is reached inside the case, the mercury contact closes a relay-circuit which breaks the heating circuit.

The only new feature in this thermostat is the following. In order to improve the regulation of the temperature and make the

heating periods shorter and more frequent, a few turns of wire—some eight or nine—in the heating circuit were wrapped directly upon the bulb of toluene. The effect of this auxiliary coil is to make the thermostat regulate every minute or two instead of every eight or ten minutes; and the effect of this more frequent regulation is, in turn, to diminish the variation of the sensitive thermometer inside the case from two- or three-tenths to two- or three-hundredths of a degree.

TABLE I

Time	Temperature inside Thermostat	Temperature of Room Outside
December 22, 1923		
8:30 A.M.	26°.14	22°.4
9:00	25.98	.....
9:45	25.95	22.4
1:00 P.M.	25.99	22.6
2:00	26.05	.....
2:50	26.07	.....
3:05	26.08	.....
3:30	26.03	.....
4:12	26.04	23.0
4:50	26.05	23.2
4:55	26.06	.....
8:00	25.95	22.5
December 23, 1923		
10:23 A.M.	25°.96	21°.5
12:20 P.M.	25.95	21.5
3:46	25.97	20.3
5:35	25.98	20.0
8:53	26.00	19.7
December 24, 1923		
9:00 A.M.	26°.14	19°.5
11:00	26.12	19.7
11:40	26.11	19.9
1:40 P.M.	26.11	20.0
2:35	26.11	20.1
3:22	26.10	20.3
4:43	26.10	20.3
December 25, 1923		
9:00 A.M.	26°.10	19°.5
1:20 P.M.	26.13	18.9
5:10	26.14	18.2
December 26, 1923		
8:10 A.M.	26°.25	18°.0
11:00	26.25	18.5
1:08 P.M.	26.10	18.3
4:45	26.10	18.6
December 27, 1923		
8:10 A.M.	26°.22	19°.2
11:45	26.19	19.5
1:20 P.M.	26.18	19.7
4:50	26.17	20.0
December 28, 1923		
8:05 A.M.	26°.17	21°.7
3:00 P.M.	26.10	21.3
5:10	26.08	21.3
December 29, 1923		
8:10 A.M.	26°.09	21°.5
12:20 P.M.	26.07	22.0

The foregoing thermometer readings (Table I), covering a period of one week, will show the degree of constancy obtainable without giving the thermostat any attention whatever. The variations of temperature in the interior of the case from one part to another are

perhaps quite as large as the time-variations at any one point. But the variations of temperature in the grating—a large plate of metal nearly an inch thick—are in all probability quite below 1/100° C. It appeared, therefore, unnecessary to introduce artificial circulation of the air inside the thermostat-case.

MEASUREMENT OF WAVE-LENGTHS[1]

The method by which the wave-lengths were determined is the well-known one of interpolating the titanium lines between the "known" wave-lengths of the iron-arc at atmospheric pressure. This is done by use of a most excellent screw for which I am indebted to our university instrument maker, Mr. F. Küng. The screw has a pitch of 1.73 mm. It was cut and ground at constant temperature and upon test was found so nearly perfect as not to require any correction for this work. Each spectrum plate was measured at least twice, once from red to blue and once from blue to red. As soon as the readings on the measuring engine were completed, a correction curve was drawn in which the following lines were used as standard:

1. The iron lines given by Keivin Burns—*Lick Observatory Bulletin*, No. 247, 29, 1913—corrected to agree with the interferometer measures given in *Scientific Papers of the U.S. Bureau of Standards* (Nos. 251 and 274, 1916).

2. The iron lines by St. John and Babcock—*Astrophysical Journal*, **53**, 269, 1921.

3. The titanium lines given by F. L. Brown—*Astrophysical Journal*, **56**, 53, 1922.

These values being plotted as ordinates on a scale of wave-lengths as abscissae, three similar, but not identical, series of points were obtained. A smooth curve drawn through these points enables one to obtain a correction for each titanium line. The mean of the wave-lengths thus obtained for each line is given in terms of the international angstrom in the following table. The second column gives the intensities on a scale running from 1 to 10: where "1" indicates a line just easily visible; "000" denotes a line which is

[1] The bibliography of previous measurements on this spectrum is given with practical completeness in Kayser's *Handbuch der Spectroscopie*, **6**, 655. The wave-lengths obtained by the principal observers, Hasselberg, Rowland, Fiebig, Kilby, Exner, and Haschek, are there given *in extenso*.

TABLE II

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
3653.499...	4	1		3827.482...	0	1	
3658.130...	2	1		3827.644...	1	1	
3671.679...	2	3		3828.020...	0	2	
3685.197...	7	3		3828.180...	2	3	
3685.964...	0	1		3829.727...	0	2	
3689.909...	1	3		3833.186...	1	3	
3694.445...	1	1		3833.674...	1	3	
3700.055...	0	1		3833.919...	1	2	
3702.291...	1	1		3836.085...	1	2	
3702.942...	∞	1		3836.534...	0	1	
3704.295...	1	2		3836.763...	2	3	
3706.219...	2	1		3842.612...	0	1	
3707.549...	0	1		3842.985...	0	1	New *Ti* ?
3708.625...	0	1		3844.234...	∞	1	New *Ti* ?
3709.963...	2	2		3845.125...	∞	1	
3715.371...	0	1		3846.438...	3	2	
3717.397...	2	3		3848.307...	0	1	
3721.632...	2	3		3850.516...	0	1	New *Ti* ?
3722.568...	1	4		3852.389...	2	2	New *Ti* ?
3724.566...	3	4		3853.038...	2	3	
3725.155...	2	4		3853.719...	3	3	
3729.806...	6	4		3855.622...	0	1	New *Ti* ?
3733.767...	∞	1		3856.657...	0	1	New *Ti* ?
3735.660...	0	1		3856.922...	0	1	New *Ti* ?
3738.995...	1	1		3857.691...	∞	1	New *Ti* ?
3741.058...	5	4		3857.901...	2	1	
3741.633...	4	3		3858.133...	3	3	
3748.010...	1	1		3860.625...	0	1	
3748.101...	0	2		3861.079...	1	1	
3752.861...	7	3		3861.549...	0	1	
3753.623...	3	3		3862.489...	∞	1	New *Ti* ?
3757.684...	2	2		3862.823...	2	3	
3759.294...	7	4		3864.302...	∞	1	New *Ti* ?
3761.322...	7	4		3864.500...	1	1	
3761.866...	0	3		3866.023...	1	1	
3766.445...	0	1		3866.441...	3	3	
3771.651...	3	4		3867.739...	2	1	
3776.062...	0	2		3868.397...	3	3	
3786.040...	3	4		3869.275...	2	1	
3786.253...	0	2		3869.615...	2	1	
3789.293...	1	4		3870.129...	1	1	
3795.903...	1	1		3871.374...	2	1	
3796.899...	1	1		3873.203...	2	2	
3798.308...	1	2		3874.124...	1	2	
3801.093...	0	1		3875.261...	3	3	
3811.385...	0	1		3877.591...	1	2	
3813.261...	0	1		3878.903...	1	1	New *Ti* ?
3813.390...	0	1		3882.147...	4	3	
3814.580...	3	2		3882.313...	3	3	
3814.855...	1	1		3882.892...	6	3	
3817.639...	1	1		3884.090...	0	2	
3818.204...	1	1	New *Ti*?	3886.228...	2	1	New *Ti* ?
3822.026...	2	2		3888.020...	2	3	

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
3889.948	1	3		4003.789	2	3	
3895.244	5	3		4005.952	1	2	
3897.290	3	1		4006.307	0	2	New *Ti* ?
3897.581	0	2		4007.195	0	1	
3898.487	1	2		4008.046	2	2	
3899.668	2	1	New *Ti* ?	4008.916	6	2	
3900.543	8	3		4009.653	2	2	
3900.958	3	3		4012.372	2	2	
3904.780	8	3		4012.786	0	2	
3911.185	2	3		4013.576	3	2	
3912.589	0	2		4015.367	2	2	
3913.460	8	3		4016.264	1	2	
3914.332	5	3		4016.943	0	2	
3914.751	0	1		4017.757	3	2	
3915.879	∞	2		4021.812	2	2	
3916.105	0	2		4024.564	5	2	
3919.822	1	1		4026.530	3	2	
3921.420	4	3		4027.426	0	2	
3924.526	8	3		4028.332	1	2	
3926.319	2	3		4030.503	4	2	
3929.871	7	3		4031.753	1	2	
3932.007	2	3		4032.628	0	1	
3933.510	1	1	*Ti* or *Ca* ?	4033.883	0	1	
3934.228	2	3		4034.884	0	2	
3938.005	0	2		4035.822	2	2	
3944.064	∞	1	*Ti* or *Al* ?	4038.314	∞	1	
3947.766	7	3		4040.310	0	2	
3948.669	8	3		4043.967	∞	1	
3956.335	8	3		4052.930	1	1	
3958.206	9	3		4053.814	1	2	
3960.967	1	1	New *Ti* ?	4055.007	4	2	
3962.852	6	3		4057.612	1	1	
3964.265	6	3		4058.134	1	2	
3968.471	1	1	*Ti* or *Ca* ?	4060.258	4	2	
3971.808	∞	1	New *Ti* ?	4064.203	3	2	
3980.821	0	2		4064.350	0	1	
3981.466	0	2	New *Ti* ?	4065.089	3	2	
3981.758	7	3		4065.595	0	1	
3981.998	1	1		4068.981	1	2	
3982.478	5	3		4074.356	0	1	
3984.313	0	3		4076.370	0	1	
3985.246	1	3		4077.148	1	1	
3985.426	1	3		4078.466	5	2	
3985.580	1	3		4079.708	1	1	
3987.618	0	1		4082.459	6	1	
3989.581	0	1	New *Ti* ?	4099.170	3	1	
3989.756	6	3		4099.797	∞	1	*Ti* or *V* ?
3993.796	∞	2		4105.161	0	1	*Ti* or *V* ?
3994.683	0	3		4111.787	1	1	*Ti* or *V* ?
3996.648	0	2		4112.602	∞	1	New *Ti* ?
3998.633	8	3		4112.715	6	1	
3999.336	2	3		4120.037	0	1	
4002.466	2	3		4121.637	1	1	

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
4122.144...	5	1		4265.273...	1	1	
4123.287...	3	1		4265.723...	1	1	
4123.559...	4	1		4266.227...	1	1	
4127.528...	5	1		4270.139...	1	2	
4129.166...	2	1		4272.440...	1	1	
4131.244...	2	1		4273.312...	0	1	
4132.393...	00	1	New *Ti* ?	4274.408...	0	1	New *Ti* ?
4136.894...	1	1	New *Ti* ?	4274.585...	3	B	
4137.285...	4	1		4276.441...	1	2	
4142.480...	00	1		4276.657...	1	2	
4143.048...	4	1		4278.231...	1	2	
4143.280...	1	1		4278.829...	2	1	
4148.390...	00	1		4280.069...	00	1	
4149.445...	0	1		4281.384...	1	2	
4150.557...	1	1		4282.696...	3	B	
4150.809...	0	1	New *Ti* ?	4284.721...	000	1	New *Ti* ?
4150.962...	2	1		4284.988...	2	3	
4152.478...	0	1		4286.006...	5	B	
4158.277...	0	1	New *Ti* ?	4287.406...	4	B	
4158.390...	00	1		4287.893...	0	2	
4159.641...	4	1		4288.161...	0	2	
4161.524...	0	1		4289.072...	5	B	
4163.647...	3	1		4289.919...	1	2	
4164.134...	1	1		4290.222...	3	2	
4166.311...	1	1		4290.543...	2	1	New *Ti* ?
4169.330...	1	1		4290.933...	3	B	
4171.018...	2	1		4291.130...	1	2	
4171.904...	3	1		4291.214...	0	1	New *Ti* ?
4173.537...	0	1		4292.161...	0	1	*Ti* or *Mo* ?
4174.088...	00	1		4294.103...	4	B	
4174.472...	1	1		4295.751...	3	B	
4179.860...	00	1		4298.666...	4	B	
4180.498...	00	1		4299.141...	3	1	
4181.094...	00	1		4299.636...	2	3	
4183.294...	1	2		4300.052...	3	3	
4186.122...	5	2		4300.566...	5	3	
4188.694...	1	2		4301.089...	5	3	
4200.752...	1	2		4301.928...	1	2	
4203.465...	2	2		4305.474...	00	1	
4211.729...	0	2		4305.910...	6	B	
4224.795...	1	2		4307.862...	4	1	
4227.654...	1	2		4308.514...	0	1	
4237.786...	0	2		4309.071...	0	1	
4237.895...	1	2		4310.373...	00	1	
4241.441...	000	1	New *Ti* ?	4311.654...	0	1	
4249.127...	1	1		4312.866...	2	2	
4251.618...	1	1		4314.356...	1	2	
4251.769...	1	1		4314.800...	4	B	
4256.027...	3	2		4314.979...	1	1	
4258.543...	1	1		4316.679...	000	1	New *Ti* ?
4260.738...	00	1		4316.807...	00	1	
4261.609...	1	2		4318.630...	2	B	
4263.134...	4	B		4320.965...	0	1	

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
4321.652...	3	B		4432.599...	1	1	
4322.220...	1	2	*Ti* or *Tb* ?	4433.578...	1	1	
4323.440...	00	1		4434.001...	$d\{^{2}_{3}\}$	3	Double
4325.132...	3	3		4434.369...	00	1	
4326.359...	2	3		4436.591...	2	2	
4326.986...	000	1		4438.232...	1	1	
4330.264...	00	1		4440.345...	2	B	
4330.708...	00	1		4441.272...	1	2	
4334.840...	00	1		4443.044...	1	1	
4337.915...	4	B		4443.799...	3	B	
4338.476...	000	1		4444.267...	1	1	
4341.369...	0	1		4444.559...	0	1	
4343.798...	000	1		4449.143...	4	B	
4344.291...	1	1		4449.985...	1	2	
4346.110...	2	1		4450.487...	1	2	
4354.064...	1	1		4450.893...	4	B	
4360.489...	2	2		4453.308...	4	B	
4367.657...	1	1		4453.707...	4	4	
4368.941...	00	1		4455.319...	4	B	
4369.685...	2	2		4457.427...	5	B	
4372.380...	1	1		4462.099...	1	1	
4374.825...	00	1		4463.391...	1	4	
4386.858...	1	1		4463.539...	1	4	
4388.077...	1	1		4464.458...	00	1	
4388.549...	00	1		4465.807...	2	B	
4390.977...	1	1		4468.493...	3	B	
4393.928...	4	2		4470.864...	000	1	
4394.057...	00	1		4471.239...	2	B	
4395.036...	5	B		4474.856...	1	3	
4395.848...	0	1		4479.724...	1	1	
4399.770...	1	3		4480.600...	0	1	
4404.276...	2	B		4481.259...	3	B	
4404.397...	1	1		4482.692...	1	1	
4404.911...	1	1		4485.013...	0	1	
4405.694...	0	1		4488.319...	0	1	
4411.080...	1	1		4489.089...	2	B	
4412.436...	000	1		4492.540...	000	1	
4415.581...	00	1		4495.006...	0	1	
4416.535...	1	3		4496.146...	2	B	
4417.275...	2	B		4497.709...	000	1	
4417.718...	2	3		4501.269...	3	B	
4418.340...	0	1		4503.759...	0	2	
4421.460...	0	1		4506.338...	00	1	
4421.756...	2	3		4511.176...	1	2	
4421.949...	00	1		4512.734...	4	B	
4422.821...	2	3		4515.610...	000	1	
4424.401...	1	1		4518.024...	5	B	
4425.840...	1	1		4518.700...	1	3	
4426.055...	2	3		4522.797...	4	B	
4427.098...	5	B		4527.305...	3	B	
4430.023...	1	1		4533.243...	6	3	
4430.368...	2	1		4533.333...	0	1	New *Ti* ?
4431.284...	2	1					

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
4533.966...	4	3		4734.682...	∞∞	1	
4534.782...	6	3		4742.129...	∞	1	
4535.574...	5	3		4742.790...	5	B	
4535.920...	4	3		4747.680...	1	2	
4536.051...	4	3		4758.125...	6	2	
4537.224...	∞	1		4758.913...	0	2	
4539.096...	0	3		4759.271...	6	2	
4544.689...	4	B		4766.330...	0	2	
4548.094...	∞	2		4769.775...	0	2	
4548.767...	4	B		4771.103...	0	1	
4549.624...	4	3		4778.263...	3	B	
4552.454...	5	B		4779.986...	0	2	
4555.069...	0	2		4781.718...	1	2	
4555.485...	5	B		4792.484...	3	B	
4557.857...	∞	2		4796.210...	2	2	
4558.092...	∞	2		4796.472...	0	1	New *Ti* ?
4559.926...	1	3		4797.983...	1	2	
4562.637...	0	2		4799.804...	3	2	
4563.427...	1	3		4805.105...	2	2	
4563.757...	3	B		4805.418...	4	B	
4570.906...	0	2		4808.531...	2	2	
4571.970...	5	B		4811.074...	1	2	
4589.961...	0	2		4812.240...	1	2	
4599.223...	1	3		4819.039...	1	2	
4609.361...	∞∞	2		4820.412...	5	B	
4617.270...	5	B		4825.445...	0	2	
4621.136...	0	1		4832.065...	0	1	
4623.098...	4	B		4836.125...	2	2	
4629.337...	2	B		4840.874...	7	B	
4637.887...	2	3		4843.989...	1	1	
4639.369...	2	3		4848.487...	2	2	
4639.669...	2	3		4854.727...	∞	1	New *Ti* ?
4639.944...	2	3		4856.012...	6	B	
4645.194...	1	B		4857.461...	∞∞	1	New *Ti* ?
4650.016...	1	B		4864.187...	2	1	
4655.712...	∞	1		4868.261...	5	B	
4656.048...	0	1		4870.129...	5	B	
4656.468...	5	B		4874.025...	∞	2	
4667.585...	4	B		4880.922...	0	2	
4675.121...	1	2		4882.326...	0	2	
4681.909...	6	B		4885.082...	6	B	
4688.392...	∞∞	1		4891.828...	0	1	
4690.827...	∞∞	1		4893.065...	1	1	
4691.334...	1	B		4893.412...	1	1	
4693.670...	∞	1		4899.911...	6	B	
4696.923...	∞	1		4903.734...	0	1	New *Ti* ?
4698.763...	2	B		4909.105...	0	1	
4710.187...	2	B		4911.205...	0	2	
4715.307...	0	1		4913.617...	5	B	
4722.603...	1	B		4915.236...	0	2	
4723.171...	1	B		4915.749...	∞	1	New *Ti* ?
4731.162...	1	2		4919.863...	2	3	
4733.426...	0	2		4921.768...	3	3	

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
4925.396...	0	3		5058.297...	∞	1	New *Ti* ?
4926.148...	0	2		5062.113...	1	2	
4928.339...	4	B		5064.068...	2	1	
4937.719...	1	2		5064.656...	7	B	
4938.291...	3	B		5065.985...	1	2	
4941.562...	0	1		5068.332...	∞	2	
4948.183...	1	1		5069.351...	∞	2	
4964.713...	1	2		5071.475...	1	B	
4968.566...	1	3		5073.139...	∞	1	New *Ti* ?
4973.051...	2	3		5085.333...	∞	2	
4973.431...	0	1	New *Ti* ?	5087.055...	2	2	
4975.345...	3	3		5104.094...	0	1	
4977.731...	1	2		5109.427...	1	1	
4978.193...	3	3		5113.444...	3	B	
4981.734...	8	B		5120.419...	4	B	
4982.716...	0	1	*Ti* or *W* ?	5122.082...	∞	1	
4989.140...	3	3		5129.143...	1	1	
4990.072...	0	1		5132.931...	0	1	
4991.065...	8	B		5145.463...	4	B	
4995.062...	0	1		5147.487...	3	B	
4997.102...	2	3		5152.185...	3	B	
4998.874...	0	1	New *Ti* ?	5154.061...	1	1	
4999.502...	7	B		5173.744...	7	B	
5000.991...	3	2		5186.329...	0	1	
5007.211...	7	B		5192.970...	6	B	
5007.873...	2	1	New *Ti* ?	5203.572...	∞	1	New *Ti* ?
5009.651...	1	2		5206.059...	4	2	
5013.284...	2	2		5207.852...	1	1	
5014.195...	3	1		5210.386...	8	B	
5014.283...	5	B		5212.271...	1	1	
5015.564...	1	1		5212.997...	0	1	
5016.165...	5	B		5218.090...	1	1	
5020.023...	6	B		5219.092...	1	1	*Ti* or *Nb* ?
5022.864...	6	B		5219.697...	2	2	
5024.831...	5	B		5222.685...	2	1	
5025.589...	4	B		5223.623...	2	1	
5027.490...	0	1	*Ti* or *U* ?	5224.304...	3	B	
5029.985...	0	1	New *Ti* ?	5224.558...	2	1	
5031.265...	∞	1	New *Ti* ?	5224.946...	3	B	
5031.534...	∞	1	New *Ti* ?	5226.534...	1	1	
5035.907...	7	2		5230.967...	0	1	
5036.462...	7	2		5238.560...	3	1	
5038.399...	7	2		5246.143...	1	1	
5039.953...	7	B		5246.619...	2	1	
5040.642...	1	2		5247.293...	2	1	
5043.578...	1	1		5252.105...	2	2	
5044.296...	1	1		5255.811...	1	B	
5044.867...	0	1	New *Ti* ?	5257.583...	∞	1	
5045.400...	0	1		5259.976...	1	2	
5046.823...	0	1	New *Ti* ?	5263.483...	1	2	
5052.879...	1	2		5265.971...	2	B	
5054.070...	1	2		5271.603...	0	2	
5056.210...	000	1	New *Ti* ?	5282.378...	1	2	

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
5283.441	2	B		5499.310	0	1	New *Ti* ?
5284.380	0	2		5503.327	1	1	New *Ti* ?
5288.798	1	2		5503.896	4	3	
5295.784	1	2		5510.162	1	1	
5297.229	2	2		5511.795	2	4	
5298.422	2	2		5512.526	6	B	
5300.012	0	1		5514.350	5	B	
5313.239	1	1		5514.536	6	B	
5336.809	1	1		5521.725	1	1	*Ti* or *Sr* ?
5341.492	0	2		5523.561	1	1	
5344.412	0	1	New *Ti* ?	5523.734	1	1	
5351.072	2	3		5527.606	1	2	*Ti* or *V* ?
5366.651	0	3		5530.449	2	2	
5369.646	2	4		5530.885	1	2	
5381.020	1	1		5537.327	0	1	New *Ti* ?
5389.180	1	4		5537.501	0	1	New *Ti* ?
5389.993	2	4		5540.963	0	1	
5396.728	0	1		5541.875	0	1	
5397.126	1	4		5553.333	1	1	
5409.614	2	B		5565.477	3	B	
5418.802	1	1		5570.532	0	1	
5419.189	1	1		5579.159	1	2	
5422.385	0	1		5582.979	2	2	
5426.256	1	4		5585.666	2	2	
5429.148	2	4		5597.689	3	3	Head of fluting
5432.318	0	1	New *Ti* ?				
5435.849	0	1		5600.795	2	1	In fluting *Ti* ?
5436.703	0	3					
5438.310	1	3		5612.689	1	1	In fluting *Ti* ?
5446.593	1	4					
5447.926	0	1		5629.279	0	1	In fluting
5448.882	0	3		5630.873	2	1	
5449.155	0	3		5635.840	2	1	
5449.556	000	1	*Ti* or *Ir* ?	5641.685	0	1	
5451.965	1	1	New *Ti* ?	5642.789	0	1	
5453.646	1	4		5644.136	5	B	
5460.502	2	3		5648.565	3	3	
5471.198	2	3		5662.155	4	B	
5472.696	1	3		5662.891	3	3	
5473.517	1	1	*d*	5667.584	2	2	
5474.228	2	3		5673.052	1	2	
5474.449	1	2		5673.424	2	2	
5477.697	5	B		5675.413	3	B	
5481.421	4	3		5679.908	3	3	
5481.862	2	3		5684.720	1	1	New *Ti* ?
5482.237	1	1	New *Ti* ?	5689.461	3	3	
5488.193	4	3		5702.666	3	3	
5490.153	4	3		5703.900	0	1	
5490.840	0	1		5708.199	1	3	
5491.283	0	1	New *Ti* ?	5711.852	2	3	
5491.656	1	2		5713.895	2	2	
5492.779	0	1	New *Ti* ?	5715.127	3	3	
5494.726	1	1	New *Ti* ?	5716.450	2	3	

TABLE II—*Continued*

Wave-lengths in I.A.	Intensities	Number of Plates	Notes	Wave-lengths in I.A.	Intensities	Number of Plates	Notes
5720.445...	2	3		5965.825...	3	B	
5727.064...	ooo	1		5978.541...	2	B	
5739.484...	2	3		5999.686...	1	2	
5739.975...	2	3		6064.631...	oo	2	
5741.192...	1	1		6085.224...	1	B	
5756.852...	1	2		6091.172...	1	B	
5762.295...	2	3		6092.733...	oo	1	
5766.339...	2	3		6098.655...	o	3	
5774.038...	3	3		6126.213...	1	B	
5780.778...	1	1		6138.723...	oo	2	New *Ti* ?
5785.990...	3	3		6199.636...	ooo	1	New *Ti* ?
5797.445...	o	2		6215.212...	2	2	
5804.264...	3	B		6220.460...	2	2	
5812.827...	1	1		6221.291...	o	2	
5823.679...	1	1		6258.099...	5	B	
5838.539...	1	1	New *Ti* ?	6258.700...	5	B	
5866.451...	4	B		6261.097...	5	B	
5899.292...	3	B		6303.774...	o	2	
5918.542...	1	2		6312.265...	oo	2	
5922.109...	1	B		6336.111...	o	1	
5941.746...	o	2		6366.382...	oo	1	
5953.160...	2	B					

barely visible or visible only with averted vision; "o" and "oo" signify intensities lying between "1" and "ooo." In the third column certain lines are marked "B"; this is to indicate those lines which Professor Brown has measured by the Fabry and Perot interferometer and which have been used as standards.

Several years ago when about to instal a spectrograph of large dispersion I was in doubt as to whether the plane grating mounted in Littrow style or the curved grating mounted in Rowland's way was better adapted to the measurement of wave-lengths through large ranges.

Having no longer any doubts upon this subject, it may be worth while to set down some of the considerations which weigh for and against these two forms of spectrograph.

In favor of the Littrow are:

1. Its simplicity of mounting: a single iron beam attached to a firm wall or two piers without any beam—small space in either case.

2. The brightness of the spectra in orders higher than the first; the width of the spectrum does not increase with the order as in the curved grating.

3. The relatively fixed position of source, grating, and camera—thus permitting easy temperature control.

In favor of the curved grating are:

1. The absence of any collimating lens, thus permitting the use of the instrument in the shorter wave-lengths of the ultra-violet, and preventing the loss of intensity which a beam suffers in crossing eight optical surfaces.

2. The absence of any adjustment for azimuth of the photographic plate which stands always at right angles to the diagonal of the Rowland mounting. This means a great reduction in the number of exposures required and a corresponding economy of time.

3. The absence of any adjustment for focus, the focal length being *practically* the same for all wave-lengths, thus avoiding the long preliminary determination of the color-curve of a collimating lens.

4. The fact that the dispersion is practically constant for all wave-lengths and that the corrections to be applied during the process of interpolation between standard lines are much smaller and more easily handled than in the case of the Littrow.

SUMMARY

1. More than 700 of the known lines of the titanium vacuum-arc and some 78 new lines, all apparently belonging to titanium, have been measured.

2. An improvement upon the ordinary toluene thermostat is described.

3. Merits of the plane and concave grating for wave-length determinations are compared.

4. The impurities found in cast titanium are practically nil. The titanium bands disappear at low pressure, as has already been pointed out by A. S. King (*Astrophysical Journal*, **39**, 139, 1914). Only traces of two bands were observed.

I take pleasure in acknowledging my indebtedness to Mr. George L. Schnable, a very skilful and reliable observer, now with the Western Electric Company, for assistance both in photographing and in measuring a considerable number of the wave-lengths in the table given above.

NORTHWESTERN UNIVERSITY
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THE EXCITATION OF THE SECONDARY SPECTRUM OF HYDROGEN BY ELECTRON IMPACTS

By O. S. DUFFENDACK

ABSTRACT

Methods of excitation.—The secondary spectrum of hydrogen was excited by two methods: (1) in a low-voltage arc in a mixture of mercury vapor and hydrogen, and (2) by electron impacts in a three-element tube. The excitation of the spectrum below the ionization potential, 16 volts, was studied in particular, and it was found that practically the entire spectrum developed below this voltage. The behavior of the *Fulcher bands and other groups* was observed and differences noted. The first two Balmer lines were successively excited at their radiating potentials. The presence of monatomic hydrogen in the mercury mixture is accounted for by impacts of the "second kind" with excited mercury atoms.

Results of the experiments.—The conclusion is reached that there must be a *characteristic ultra-violet spectrum* due to the molecule whose initial state is identical with the final state of the molecule after the emission of the secondary lines. This conclusion is supported by spectroscopic investigations in the extreme ultra-violet and by investigations of critical potentials in hydrogen. The results indicate that the whole secondary spectrum is due to the neutral molecule and that its excitation is not dependent upon the dissociation of the molecule. The minimum ionization potential is about 16 volts at which the molecule is dissociated into a neutral atom, an ionized atom, and an electron.

Evidence from investigations of widely different character in the past three years leaves little room for doubt that the series spectra of hydrogen are emitted by the atom while the secondary spectrum is due to the molecule. It might be well to mention several of these investigations by way of summarizing the evidence. In 1920 Mr. G. P. Thomson[1] showed that the series lines were strongest in the emission from positive rays in hydrogen when the atomic parabola was strongest on his fluorescent screen, and the secondary lines were strongest when the molecular parabola predominated. In 1921 Gehreke and Lau[2] showed that the Doppler effect for the Balmer lines was $1/2$ times as great as for lines of the secondary spectrum. Merton and Barratt[3] in 1922, by more exact methods than had been previously used, showed that the mass of the emitters of the secondary lines, as determined from the width of the spectral lines, was that of the molecule. The failure of the

[1] *Philosophical Magazine*, **40**, 240, 1920.

[2] *Zeitschrift für Instrumentenkunde*, **41**, 100, 1921.

[3] *Phil. Trans. Roy. Soc.*, A, **222**, 369, 1922.

lines of the secondary spectrum to show in the spectra of the sun and of stars when the Balmer lines appear is explained by the dissociation of the gas at the temperatures and pressures prevailing. The same results were obtained by the writer[1] when hydrogen was dissociated by thermal action in a tungsten furnace.

While the series laws of the primary spectrum are well known, very little progress has been made toward the analysis of the secondary spectrum. Early investigations of the excitation of the spectrum in vacuum tubes under varied conditions of pressure and with and without a condenser or spark gap in series showed some variations in the relative intensities of the lines, but no basis for an analysis of the spectrum appeared. In 1906 Dufour[2] investigated the strong lines of a limited region for the Zeeman effect, and found that some lines showed the effect and some did not. This indicated that there are at least two classes of lines in the spectrum. The Stark effect has been investigated for the strongest lines by various observers,[3] but no basis for a separation of the lines into groups is apparent in the results.

In 1913 Fulcher[4] investigated the excitation of the secondary lines by electron impacts at high and low voltages. He found that some of the lines in the region λ 6400–λ 5400 increased in intensity when the voltage was raised, while most of the lines showed little change. He used this as a basis for the first classification of the lines into groups, putting the lines that are unchanged into group I and the lines that are more intense at high voltages into group II. The group I lines are, in general, low-pressure lines and show no Zeeman effect, and the lines of group II are often high-pressure lines, strong in a condensed discharge, and show the Zeeman effect. He also pointed out[5] that some of the lines of group I have constant wave-length differences resembling bands. One group of these lines lies in the red and yellow and another group in the green. Kiuti[6]

[1] *Physical Review*, **20**, 665, 1922.

[2] *Ann. de Chem. et de Phys.*, **9**, 36, 1906.

[3] Takamine and Yoshida, *Mem. Coll. of Sci., Kyoto*, **2**, 321, 1917; Nitta, *ibid.*, **2**, 349, 1917; Takamine and Kokuba, *ibid.*, **3**, 271, 1919; Kiuti, *Jap. Jour. of Physics*, **1**, 29, 1922.

[4] *Astrophysical Journal*, **37**, 60, 1913.

[5] *Loc. cit.*, and *Physical Review*, **21**, 375, 1923.

[6] *Phys. Math. Soc., Japan*, **5**, 9, 1923.

has recently found another group in the orange which, he says, corresponds to the null branch of a set in which the Fulcher bands are the positive and negative branches. Hughes[1] investigated the variation of the intensity of a number of lines with the energy of the exciting electrons from 29 to 110 volts, and found that the Fulcher band lines decrease in intensity continuously from the lowest voltage while other lines, three of which show the Zeeman effect but are beyond Fulcher's classification, give a maximum density on the photographic plate between 30 and 40 volts, and fall off in intensity less rapidly than the Fulcher lines.

Merton[2] in 1919 and later with Barratt[3] studied the lines of the secondary spectrum excited in a vacuum-tube discharge in a mixture of hydrogen and helium. They found that some of the lines are apparently unaffected by the helium, others are somewhat enhanced in the mixture, and some are greatly enhanced. A number of lines were found that had never been observed in pure hydrogen. Barratt[4] has recently reported a similar but weaker effect of argon. The high-pressure, condensed discharge lines that fall in Fulcher's group II are usually enhanced in the helium mixture, but no general agreement between Fulcher's classification and the results of these investigations is evident.

The net result of all these experiments is that there are certainly two or more different classes of lines in the secondary spectrum and that some of the lines are associated together into bands.

The present investigation was undertaken to continue the study of the excitation of the secondary spectrum by electron impacts to lower voltages than had been investigated. It was desired, in particular, to determine whether this spectrum developed below the ionization potential of hydrogen, about 16 volts, and whether this could be excited without exciting the series lines. It was hoped that the investigation would reveal whether or not there was any connection between the excitation of this spectrum and the dissociation of the molecule and also the electrical condition of the molecule at the time of emission. Nearly all of these purposes were accomplished, as will be shown in the discussion of the results.

[1] *Physical Review*, **21**, 292, 1923.
[2] *Roy. Soc. Proc.*, A, **96**, 382, 1919.
[3] *Loc. cit.*
[4] *Philosophical Magazine*, **46**, 627, 1923.

THE APPARATUS

Two methods of excitation were employed: (1) low-voltage arcs in mixtures of mercury vapor and hydrogen for observation below the ionizing potential of hydrogen and in hydrogen alone above the ionizing potential, and (2) excitation by electron impacts at low pressures in a three-element tube.

The discharge tube for method 1 contained an incandescent filament cathode, tungsten or molybdenum, and a nickel-plate anode placed about 7 mm from the cathode. Redistilled mercury was introduced into the tube when it was desired to use the mixture, and the tube was then baked out thoroughly. The mercury in the tube distilled into a condenser at the end of the tube during the baking and ran back into the tube. In this way the mercury was further purified and at the same time its vapor acted as a pump to sweep out gases liberated from the walls of the tube and from the electrodes which were glowed out at the same time. The tube was kept in the baking oven during the run of the experiment, and the mercury-vapor pressure was regulated by controlling the temperature of the oven.

By regulating the mercury-vapor pressure and the pressure of the hydrogen, arcs could be maintained at any desired voltage down to 5.5 volts. However, when the mercury-vapor pressure was great enough to permit an arc to be struck at or less than the ionizing potential of mercury, 10.14 volts, there was a noticeable limiting of the electron speeds, probably to 5.5 volts. This is due to a relatively high concentration of mercury atoms in the $2p_2$ or other metastable state as a result of the absorption of resonance radiation or other cumulative action. Under these conditions no trace of any hydrogen lines was found, and so this method was employed only in the region between 11 and 16.5 volts. The mercury-vapor pressure was regulated so that the arc would break at only a slightly less voltage than that at which it was desired to photograph, and when this was done there was no indication of a limiting of the electron speeds.

For the investigation of the changes in the spectrum at higher voltages, arcs in pure hydrogen were employed, and no evidence of cumulative action was manifested. This is because there was

not a sufficient amount of dissociation to give a sufficiently high concentration of monatomic hydrogen to exhibit cumulative action from atomic radiation. The molecule does not display any cumulative effects either because the life of its excited state is too short or because the difference between the energy necessary to put it into an excited state and that required to ionize it is greater than 16 volts.

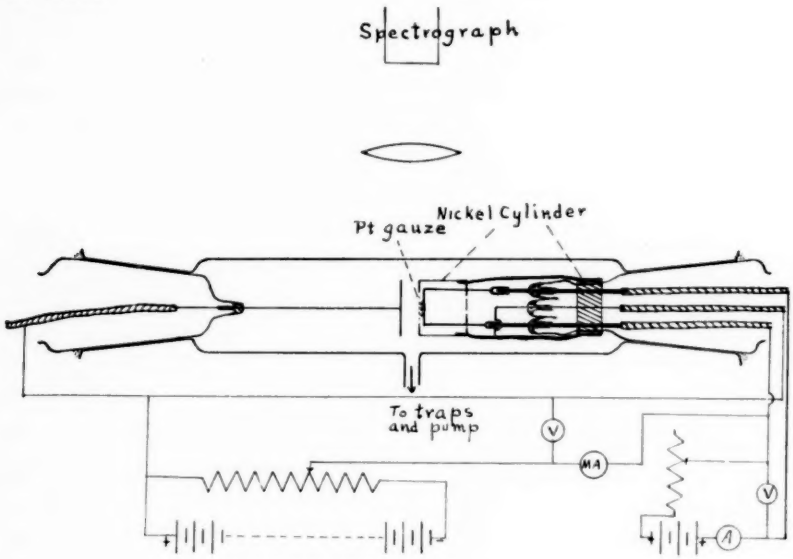

FIG. 1

The discharge tube and control circuits used in the second method are shown diagrammatically in Figure 1. This tube is made of Corning G702P glass with tungsten-wire seals. The leads carrying the filaments were of 100-mil diameter, and conducted a current of 35 amp. without undue heating. The stems were connected to the discharge tube by ground-glass joints. No grease or lubricant was put on these joints, but a little De Khotinsky cement was put around the ends to insure their being tight. The cathode usually consisted of a grid of tungsten filaments. Surrounding the filaments was a nickel cylinder which was closed at one end except for a window of platinum gauze. An accelerating field was applied between the filaments and the gauze which was mounted

as close to them as possible in order to get a large thermionic current without the necessity of striking an arc. The nickel cylinder acted as a light as well as an electrical shield. The filaments and cylinder were mounted on the same removable stem and replacements and adjustments could be made outside the tube. Thus it was not necessary to change the adjustment of the spectrograph during the investigation. A nickel-plate anode was in metallic contact with the gauze, and so the electrons passed into a force-free region with velocities regulated by the accelerating field applied between the filaments and gauze. The difference of potential between the gauze and the middle of the filaments was taken as the exciting voltage. The tube was inserted into an electric oven as far as the ground joints at the ends, and thoroughly baked out.

The hydrogen was generated by the electrolysis of dilute sulphuric acid. Before the gas was admitted to the discharge tube, it was stored for some time over phosphorous pentoxide and then passed through a liquid air trap. As the gas was adsorbed by the walls of the tube more was admitted from time to time to keep the pressure within a previously determined range. After the discharge had been running for some time, the walls approached a condition of saturation and the gas pressure changed very slowly so that it could be kept nearly constant. In the first method pressures of 0.1–1.0 mm were used, and in the second 0.01–0.5 mm.

Photographs were taken of the spectrum of the arc in the mercury mixture at 11, 12, 13, 14, and 16.5 volts, the arc current, pressure, and time of exposure being kept constant. At 11 volts the only hydrogen line to appear was $H\alpha$. No trace of $H\beta$ could be seen in this spectrogram, but it is strong at 12 volts, and both lines increase rapidly in intensity with the voltage. $H\gamma$ was obscured by the strong mercury line 4339.25, and the other members of the Balmer series were beyond the range of the spectrograph. The voltages at which these lines appeared are a little less than their radiating potentials which are 12.05 and 12.70 volts. This discrepancy is probably due to the initial velocity of emission of the electrons for which no correction was made. The successive excitation of the two lines indicates homogeneity in the electron stream, and the fact that the lines were excited at their radiating potentials supports

the view that the cathode fall of potential constitutes nearly the whole potential difference between the electrodes, as the lines were photographed near the cathode where they were strongest. This also gives one confidence in the correctness of the exciting potentials of the secondary lines which follow.

The presence of the Balmer lines in the spectrum shows that considerable dissociation of the hydrogen has taken place. This was accomplished in part by thermal action at the filament, but mostly by impacts of the "second kind" between excited mercury atoms and hydrogen molecules. Franck and Cario[1] have made a series of investigations of impacts of this type and showed, among other things, that hydrogen molecules are dissociated in this way; and Professor K. T. Compton and the writer[2] have shown that the amount of dissociation accomplished by an arc in a mixture of mercury vapor and hydrogen is much greater than that due to thermal action at the filament.

Very faint traces of some of the secondary lines could be made out with a lens on the negative of the 12-volt spectrogram, and a large number of the lines appeared strongly at 13 volts. At 14 volts, nearly the whole secondary spectrum was excited, and no noticeable change occurred when the voltage was raised to 16.5 volts. These spectrograms are reproduced in Plate II*A*.

The three electrode tubes gave similar results except that the potentials were 1 volt higher, the spectrum being practically complete at 15 volts instead of 14 volts. Only faint traces of the Balmer lines can be seen in this spectrogram, but the first three members show up strongly at 16.5 volts, Plate II*C*. The presence of a slight amount of monatomic hydrogen below 16 volts can be satisfactorily accounted for by the thermal dissociation at the filament together with dissociation by electron impacts at 15 volts[3] and by impacts of the "second kind" with excited molecules.

Two spectrograms were taken under identical conditions of pressure, arc current, and time of exposure, with hydrogen and a

[1] Franck, *Zeit. für Phys.*, **9**, 259, 1922; Cario, *ibid.*, **10**, 185, 1922; Franck and Cario, *ibid.*, **11**, 161, 1922; Cario and Franck, *ibid.*, **17**, 202, 1923.

[2] *Physical Review*, **23**, 583, 1924.

[3] Hughes, *Philosophical Magazine*, **41**, 278, 1921.

PLATE II

A
B
C
Volts
16.5
14.0
13.0
60
16.5
17
15
13
H_δ 4078
H_γ 4340
and
H_δ 4339
H_δ 4358
4339
H_δ 4347
4358
H_β 4861
H_δ 4916
H_δ 5461
H_δ 5790
5770
H_α 6563
H_δ

low mercury-vapor pressure, about 0.001 mm, at 16.5 volts and 60 volts, in order to compare the behaviors of the lines at low and high voltages. These spectrograms are reproduced in Plate II*B* and marked differences in the behavior of the lines are apparent.

Fulcher's band lines and a number of other lines of his group I appear at 13 volts in the mercury mixture, and at 14 volts these have reached their full intensity. All these lines diminish in intensity relative to others at 60 volts. Two lines of group II, λ 5916.8 and λ 5931.6, are present at 13 volts. The first of these is marked by Fulcher as of doubtful classification, but the second must certainly belong to this group as it was found by Dufour to show the Zeeman effect. A number of other group II lines appear in the 14-volt spectrogram, and none of these retain their intensity at 60 volts. Two groups of lines beyond the range of Fulcher's classification appear at 13 volts, and are of particular interest.

TABLE I

λ	I_1	I_2	H	L	C	*He*	Z
4490.45..	5	0	.....	++	.....	++	+
4498.10..	6	1	.....	.....	.....	.....	+
4508.10..	6	4	.....	++	.....	+	0
4572.73..	4	2	++	.....	.....	+	.....
4580.04..	5	4	.....	++	.....	+	0
4582.58..	4	0	.....	.....	.....	0	0
4627.96..	5	4	.....	.....	.....	0	+
4631.89..	10	3	.....	.....	.....	+	+
4633.95..	9	5	.....	.....	.....	+	+
4653.04..	5	0	.....	.....	.....	.....	+
4661.39..	4	0	.....	++	.....	.....	0
4662.79..	5	0	.....	+	.....	.....	0
4665.58..	3	0	.....	++	.....	.....	0
4719.02..	6	0	.....	.....	.....	.....	+
4723.01..	5	0	.....	.....	.....	.....	+
4838.25..	4	0	.....	.....	−	0	.....
4849.34..	6	1	.....	.....	=	0	.....
4856.54..	5	0	.....	.....	.....	0	.....
4928.62..	10	5	.....	.....	.....	.....	+
4934.29..	4	5	.....	.....	.....	.....	+

They are listed in Table I together with their behavior with respect to high, H, and low, L, pressures and condensed discharge C, in vacuum tubes; helium effect, *He;* and Zeeman effect, Z. The wave-lengths are taken from Merton and Barratt's[1] measurements,

[1] *Loc. cit.*

and their relative intensities, I_1, are given in comparison with mine, I_2. These are the lines which retain their strength in the 60-volt exposure, Plate IIC. The positive Zeeman effect for some of these indicates that they would probably belong to group II if Fulcher's classification had extended that far, and the low-pressure lines which do not show the Zeeman effect would be expected to fall into group I. If this should be true, then the early development of the former and the persistence of the latter to 60 volts in my experiments are anomalous.

DISCUSSION OF THE RESULTS

The excitation potentials for the secondary lines are too high to permit the conclusion that the radiation of any of the group of lines leaves the molecule in its normal state. Furthermore, the lack of corresponding absorption lines shows that the final state of the molecule involved in this radiation is not the normal state. It has been suggested by Sommerfeld[1] that a characteristic ultra-violet radiation is excited at about 11 volts which bears the same relation to the secondary spectrum as does the Lyman series to the Balmer series in the radiation from the atom, that is, the final state in the emission of the bands of the secondary spectum is the initial state for bands in the ultra-violet spectrum.

This investigation bears out Sommerfeld's prediction. The energy required to excite the secondary spectrum varies from about 2 to 3.5 volts, and the minimum excitation potentials are from 13 to 15 volts. This means that there should be ultra-violet radiation, requiring from 11 to 13 volts for its excitation, present simultaneously with the secondary spectrum. This ultra-violet spectrum should lie between about 900 and 1100 A, and there is spectroscopic evidence of such a spectrum. The region is the same as that occupied by the Lyman series, and lines very probably due to hydrogen and not belonging to this series are reported by Lyman[2] and by Hopfield.[3] The measurements of these lines are probably not very exact, and little can be done as yet toward putting them into groups, but if the foregoing relationship between these lines and the secondary spectrum is correct, then bands should be found in this

[1] *Atombau*, p. 422.

[2] *Spectroscopy of the Extreme Ultra-violet.*

[3] *Physical Review*, **20**, 573, 1922.

region whose initial states correspond to the final states of bands in the secondary spectrum.

Observations on the critical potentials of hydrogen likewise support this view. Critical potentials have been reported at 10.5, 11.5, 11.8, 11.9, 12.2, 12.6, 12.9 volts by various observers.[1] Some of these potentials are ascribed to the atom as a consequence of previous or simultaneous dissociation, but others are specifically assigned to the molecule. Olmstead interprets his curves to indicate radiation from the molecule from 12.1 to 13 volts, but ascribes the 11.5 potential to the ionization of the molecule without dissociation. Foote, Mohler, and Kurth report 10.5 and 11.8 volts as resonance potentials for the molecule, and Horton and Davies in their recent paper take the same view of 11.9 volts, and put the ionization potential of the molecule at 22.8 volts. We cannot expect a single radiating potential for this ultra-violet radiation if it consists of bands whose final state is the normal molecule, for then the initial state must be different for various bands. The energy required to excite each is the energy necessary to put the molecule into the required initial state. Some of the bands are probably stronger than others, and so their radiating potentials would show up most readily and give the various potentials reported. The writer's investigation of low-voltage arcs in normal hydrogen[2] does not support the contention for an ionization potential for the molecule below 16 volts. There is no indication of ionization below this voltage in the current-voltage curves for normal hydrogen. If ionization occurs at all, the degree of ionization would have to be very low for it to have no effect on the thermionic current from the filament.

The results of this investigation seem to indicate that the neutral molecule is the source of practically all of the secondary spectrum. Both of Fulcher's groups and other lines, with and without helium and Zeeman effects, are excited below 16 volts and show little change at this critical potential. If the ionization potential of the molecule is above 16 volts, as it seems to be, the neutral

[1] Franck, Knipping, and Kruger, *Verhandlungen d. Deutschen Phys. Gesellschaft*, **21**, 728, 1919; Olmstead, *Physical Review*, **19**, 532, 1922; Foote, Mohler, and Kurth, *ibid.*, p. 414; Horton and Davies, *Philosophical Magazine*, **46**, 872, 1923.

[2] *Loc. cit.*

molecule alone can be responsible for this spectrum. There are, however, undoubtedly two or more groups of lines in the spectrum, and progress in the fitting of these lines into bands is to be expected. These band groups have different initial and possibly final states, and, therefore, have different radiating potentials.

The results also seem to indicate that the dissociation of the molecule is in no intimate way connected with the excitation of the secondary spectrum. The excitation of the entire spectrum in the three-element tube at 15 volts with only traces of the Balmer lines leads to this conclusion, and the strong appearance of the Balmer lines at 17 volts supports the view that the 16-volt potential corresponds to the dissociation of the molecule into a neutral atom, an ionized atom, and an electron. Since this investigation was begun, the report of an investigation of the secondary spectrum by Keussler[1] has appeared. He used a three-element tube similar to that employed in method 2 of this investigation, and studied the excitation of pure hydrogen and of a mixture of hydrogen and helium. Keussler reports that in pure hydrogen the molecular spectrum alone is present below 16 or 17 volts, the Balmer lines completely failing, while above these voltages and up to 18 volts the Balmer lines increased in intensity relative to the many-lined spectrum. He suggests that the dissociation of the molecule may occur in two steps: first, the molecule is excited, and the second follows only if the molecule experiences a disturbance before the radiation of the acquired energy. Upon this basis he explains the relative changes in the intensities of the secondary and Balmer lines upon addition of helium. He also concludes that at the minimum ionization potential the molecule is dissociated into a neutral atom, an ionized atom, and an electron.

It seems highly desirable to investigate further the radiation from the hydrogen molecule in the extreme ultra-violet. Any other way in which the lines of the secondary spectrum can be shown to be physically related will also prove helpful in their classification. A study of the lines that are physically related will eventually lead to the analysis of this much-investigated spectrum.

PHYSICS LABORATORY
UNIVERSITY OF MICHIGAN
February 1, 1924

[1] *Zeit. für Phys.*, **14**, 19, 1923.

THE PERIODOSCOPE

By H. DE MIFFONIS

ABSTRACT

The determination of a period from a set of isolated observations is a problem of frequent occurrence, particularly in astronomy. The author describes how this problem can be solved mechanically, and complete *drawings of an instrument* for this purpose are given. It is also shown how the principles involved can be used graphically without the instrument.

At the suggestion of Dr. Henroteau, astronomer at the Dominion Observatory, Ottawa, the writer has recently endeavored to devise an instrument having for its purpose the solution of the following problem:

A periodical phenomenon of unknown period having been observed at various times, a graph is made with the times of observation in abscissa and the values of the observations (for instance the velocities of a star) in ordinate. From this graph find the most probable period of the phenomenon.

The principle, evolved by Dr. Henroteau, on which this instrument is based consists in modifying the time scale of the graph of observations until the period should have a given length arbitrarily chosen beforehand. When the proper scale is secured all the points of observation and their replica at periodic intervals form a smooth curve of a periodic nature. A few remarks concerning the probability of obtaining the right period will be better understood after a short description of the apparatus. For the latter the name "periodoscope" is suggested.

The writer wishes here to thank Mr. H. V. Anderson, mechanical engineer, Marine Department, Ottawa, for making the accompanying plans as well as for his hearty co-operation while the machine was in its experimental stage.

The instrument consists essentially of a series of parallel bars called "time bars" on which roll pins, one on each bar, are maintained in line by means of a slotted bar, the "cross arm." The time bars, which are held in place by means of four Archimedes screws, are set by the observer by means of the graduated crossbars so that

the center of the pins are at distances from the pivot of the cross arm proportional to the times of observation.

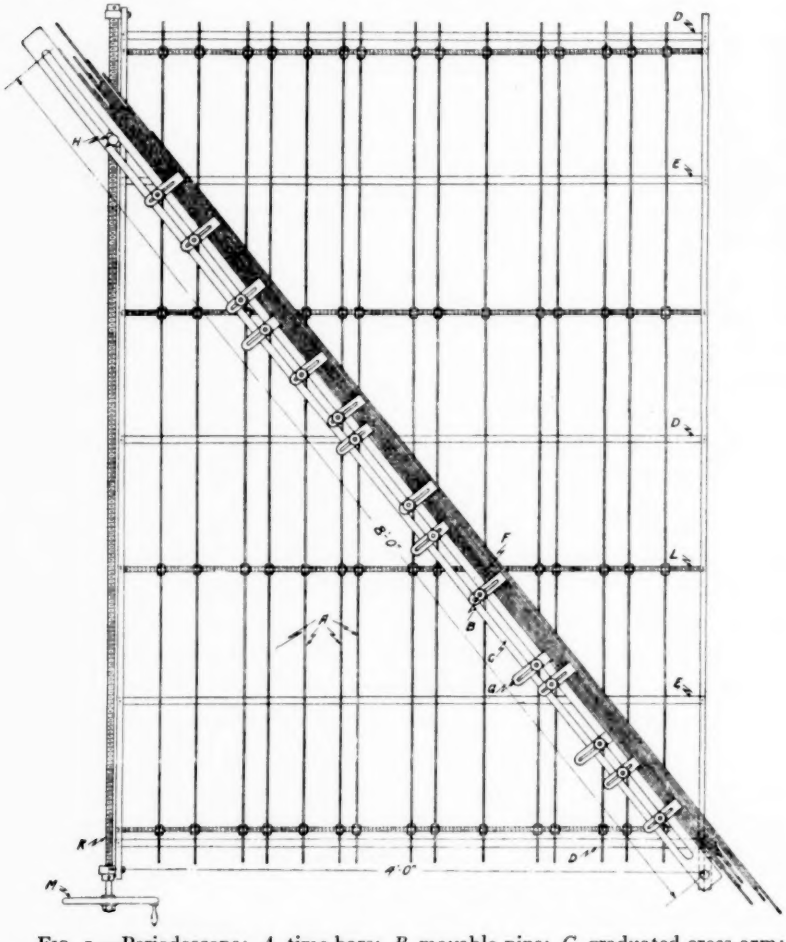

FIG. 1.—Periodoscope: *A*, time bars; *B*, movable pins; *C*, graduated cross arm; *D*, graduated crossbars; *E*, plain crossbars; *F*, period bars (wires and stirrups omitted); *G*, graduated crosspieces; *H*, movable carriage; *K*, control spindle; *L*, time-setting screws; *M*, handwheel.

Bars, *the period bars*, the length of the cross arm, with crosspieces properly placed, one for each bar, are set parallel to the cross arm. Each of these bars, by means of the graduation in the crosspiece, is connected to one of the aforesaid pins at a distance pro-

portional to the value of the observation taken at the time to which corresponds the time bar along which this particular pin moves. Above this bar, two small wires are stretched on which are mounted celluloid stirrups. These, all 2 inches long, are of two kinds, some entirely white while others have at one end a black strip $\frac{1}{8}$ inch wide. Should the latter be exclusively used, the period bars would show from above black dots 2-inch centers; if, on the contrary, n white stirrups be introduced between each black and each white one, the dots would be spaced $2(n+1)$ inches apart. This device allows thus the trial of various selected periods.

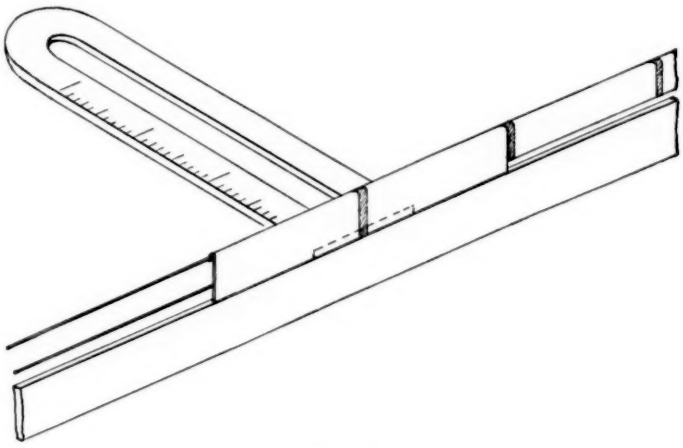

FIG. 2

The origin of the periods is, of course, for each bar, in the vertical plane perpendicular to the crossbar through the center of the pin; it is marked by a black strip on a fixed stirrup as shown on Figure 2; the celluloid stirrups are added in the proper order toward both ends of the bar.

The cross arm and the period bars are swung over the time bars on which they rest by means of a movable carriage passing through the slot of the cross arm and controlled by an Archimedes screw fitted to the side of the frame and operated by a handwheel. The crossbar progresses so that the wheels of the movable pins press against the time bars, the distances between the centers of the pins thus increase proportionally, i.e., the time scale of the graph is

altered while the scale of the observations remains the same. Since the dots representing the selected period are invariably fixed to the ordinate of observations and since these have different motions due to their various distances from the base, the respective positions of the dots vary all the time while the crossbar moves over the time bars, until, for a certain position, a smooth periodical curve may be obtained. Figure 3 shows a portion of the celluloid stirrups during the operation, and later when the proper position has been arrived at.

In order to reduce friction to a minimum each pin has been mounted on two ball bearings, one traveling in the slot of the crossbar while the other rolls against the time bar. Should, however, the friction be still found objectionable the heads of the movable

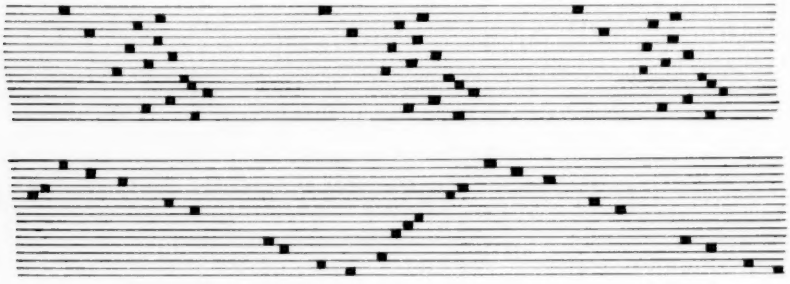

FIG. 3

pins could be connected by small coil springs in tension, the time bars being reversed and so placed that they would counteract the action of the springs and prevent the advance of the pins unless the cross arm be moved farther away from the base position.

The apparatus represented on Figure 1 has been so designed that the time scale of the graph may be doubled. The period 2 inches being at first tried, the motion of the cross arm along the time bars will give all the periods between 2 inches and 1 inch. If these have given no result the cross arm is moved back to the original position and every other black and white stirrup is replaced by a white one, thus showing a 4-inch period; the motion of the bar gives all periods between 4 and 2 inches; if unsuccessful, an 8-inch period, giving all periods between 8 and 4 inches, may be tried, and so on.

Let us assume that the period 2 inches gives a good curve, the cross arm being so placed that the index on the guiding carriage reads 0.72; the value of the proper period is thus $2 \times 0.72 = 1.44$.

It is obvious that several periods may satisfy the same graph, the fewer the observations the greater the probability; the relative positions of the observations would also have some bearing in this respect. This drawback, however, is not inherent in the instrument but would be found in any method; and it can be said in favor of the instrument that since the trials of various periods for a given set of observations entails very little labor the observer will not hesitate to investigate the whole range of possible periods, while he will rely on further properly timed observations to decide among the various periods secured.

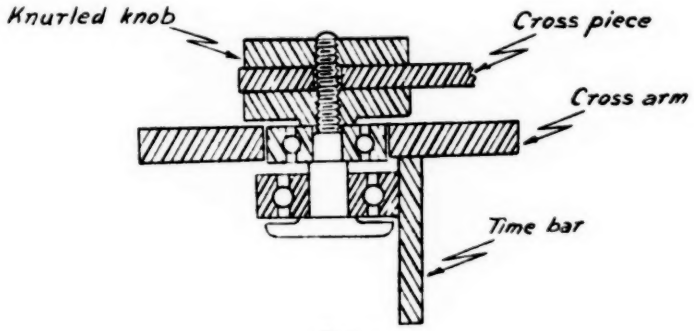

FIG. 4

In case the small number of observations would not warrant the building of a machine, a graphical method along the same lines is here suggested.

Let O be the origin, O, OA, OB . . . . OK, the times, and a, b . . . . k, the values of observation. Let us draw base lines OP, OQ . . . . at suitable intervals and the perpendiculars OM'_1, $A'M'_2$, $B'M'_3$ . . . . OM''_1, $A''M''_2$, $B''M''_3$. . . . . From the points M'_1, M'_2, ... , M''_1 . . . . we mark on parallels to the respective bases, starting from the points of the modified graphs, lengths equal to the selected period. There may be found two bases for which the period points are more smoothly arranged than the others. Subdividing the angle of these two bases as above,

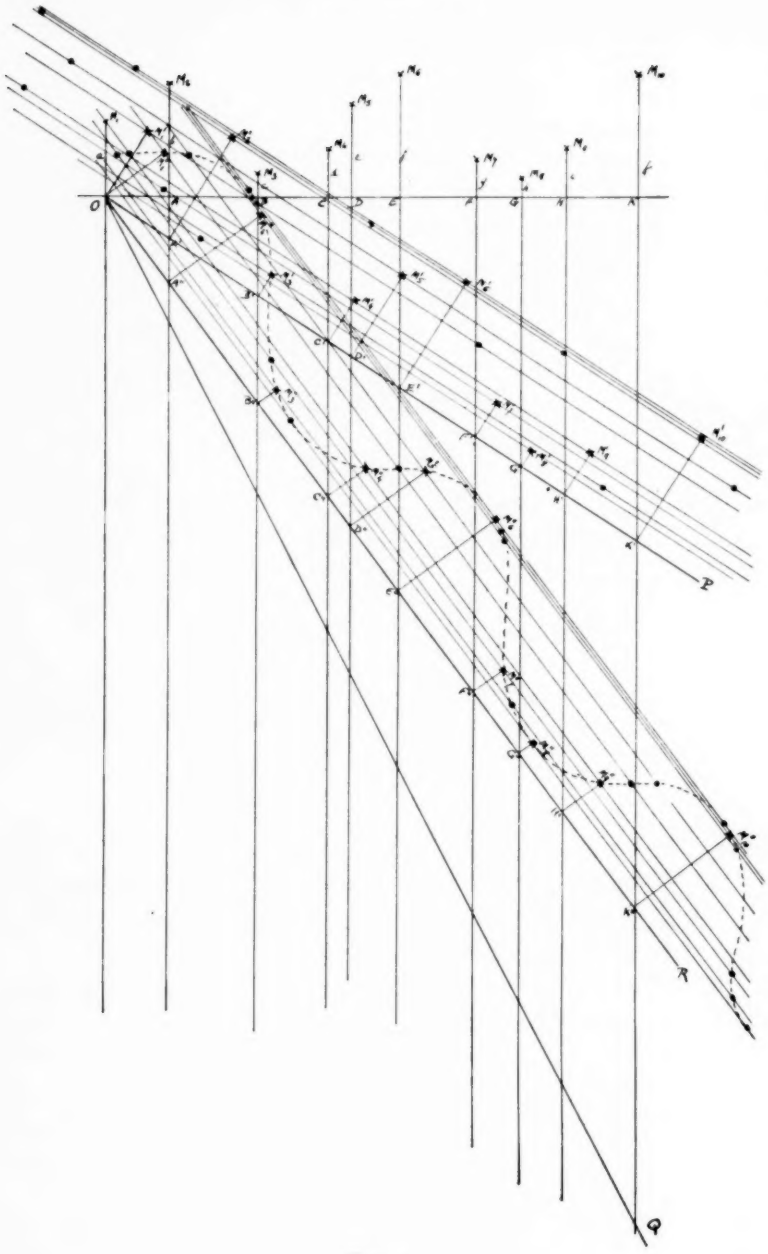

FIG. 5

a position of the base is soon found for which the period points are arranged on a smooth curve. Let L be the period selected for trial, OK'' the position of the base which gives a smooth curve, the period sought for is

$$L \times \frac{OK}{OK''}.$$

In practice, it will be found convenient to use a piece of tracing cloth with parallel lines properly spaced and divided to mark the periods and to transfer them by means of a carbon paper placed under the drawing on separate sheets; this will avoid confusion on the drawing. On the accompanying drawing, and for the same reason, only the final and one of the intermediate steps have been shown.

DEPARTMENT OF MARINE AND FISHERIES
OTTAWA, CANADA
January 15, 1924

REVIEWS

Die veränderlichen Sterne. Erster Band: Geschichtlich-Technischer Theil. VON JOHANN GEORG HAGEN, S. J. Erste Lieferung: "Die Ausrüstung des Beobachters." Zweite Lieferung: "Die Beobachtung der veränderlichen Sterne." Dritte Lieferung: "Die Berechnung der Beobachtungen." Vierte Lieferung: "Die Elemente des Lichtwechsels." Freiburg: Herdersche Verlagshandlung, 1914, 1920, 1921. 4to. Pp. xx+811.

Part I, treating of the instrumental equipment of the observer, was reviewed by the writer in this *Journal*, **40,** 483, December, 1914. What was there said in regard to the need of this work, the skill exhibited by the author, and the success of his efforts applies equally to the three parts here reviewed.

In attempting to characterize the work as a whole, it should probably be called a historic encyclopedia of variable stars. Thirty pages of Author and Subject Index at the close of Part IV permit easy reference to any subject. This review would be incomplete without a list of chapters and reference to the most striking points which they contain, so, in spite of obvious objections, this method will be followed.

Part II contains in chapter vi precautions to be considered, among them being warning in regard to the effect of twilight and haze, also the disturbing influence of star colors. Chapter vii treats of physiological errors, including the relative position in the field of view of the stars to be compared, the difficulty in estimating by Argelander's method large intervals in brightness, the effects of color and prejudice. The choice of comparison stars is considered in chapter viii, their notation and use. The fundamentals of the observations of variable stars are touched upon in chapters ix and x—the light-scale and step-value in the former, and in the latter, the three common methods of making visual comparisons: the absolute method used by Herschel, the differential (step) method of Argelander, and the decimal method used by Pogson and Pickering. The author prefers the Argelander method, implying the determination by each observer of his individual light-scale, thus giving completeness to his work and making it more valuable for reference. However, he

considers in detail the advantages of the decimal method, which implies the possession of the photometric magnitudes of the comparison stars and facilitates the combination of the estimates of many individual observers.

Chapter xi deals with star colors and their meaning in the theory of variability; the symbols used in estimating color scales, and the co-ordination of the scales employed by the principal observers, Schmidt, Osthoff, Franks, Müller and Kempf, and others. The author shows that he is a master of this highly technical division of the subject; but, adhering to his historic point of view, he does not stress the contributions made by the spectroscope to star colors. The final chapter in Part II goes into minute detail in regard to the recording of the observations, giving the learner many invaluable directions and mentioning many things which he would easily learn by experience.

The reader will find in Part III, "The Reduction of the Observations," everything needed in the preparation of his work for publication, thoroughly explained and illustrated by classical examples from the historic authorities.

Chapter xiii considers the scale derived from the comparison stars. It compares the step-scale with the magnitude-scale, and describes the use (but not the determination) of photometric magnitudes of the comparison stars.

Chapters xiv and xv deal with the light-curve derived from the observations, and the methods of deriving from it the principal points, such as maxima, minima, points of inflection, and times of greatest velocity of variation. Attention is given to the classical method of fixing the times of maxima and minima by the curve which bisects the horizontal chords of the light-curve. The author frequently uses the terms "photometric and photographic" as if contrasted, thus implying that the photographic magnitudes were merely estimated, not measured. In most cases "visual and photographic" were meant. He considers Schönfeld to be the best variable star authority, saying on page 411: "In this way Schönfeld gave his curves an accuracy which has not been equalled before or since."

Chapter xvi treats of the preparation of the reduced observations for publication, giving suggestions which may well be followed. Among these should be mentioned the necessity of furnishing complete identification for the comparison stars, and warning against publishing "raw" observations, without reducing them to at least approximate magnitudes.

Chapter xvii illustrates all the foregoing by examples of the reduction of observations of η Aquilae, ζ Geminorum, β Lyrae, Mira Ceti, χ Cygni, Algol, and S Cancri, thus including all the principal types of variation. This is a good example of the encyclopedic character of the work. It is based largely on the writings of Argelander and Schönfeld, but refers also to their most eminent successors.

Part IV: The first three parts were written for the observer, the fourth for the theoretical astronomer; excluding, however, any consideration of the causes of the phenomena. It is entitled "The Elements of the Light-Changes," and includes amplitude, epoch of the principal phases, length of period, shape of the light-curve, color and color-change, and a final chapter summarizing our knowledge of variables. After a thorough explanation of these elements in chapter xviii, the author considers the amplitude in detail in chapter xix. Knowledge of the limits of amplitude aids in the classification of variables. Varying amplitude, if not entirely irregular, leads to the discovery of composite curves. Comparison of visual and photographic amplitude points to spectral changes. The author here suggests a classification of all variables into three types: (1) Eclipsing variables, including the Algol and β Lyrae types. (2) "Blinksterne," difficult to translate in a single word but suggested by Hartwig from the likeness to intermittent illumination from a lighthouse. This includes the Cepheids, cluster types and novae. (3) "Flutsterne" which also defies translation, but suggests a pulsating ebb and flow, and includes variables of long and irregular periods.

Chapters xx, xxi, and xxii consider in detail the epoch, length of period, and shape of the light-curve, copiously illustrated by classical examples both of graphic and algebraic reduction. Some diagrams would be valuable, but none are given.

Chapter xxiii, on color and color-changes, is of special interest, since it combines with the historic treatment some of the results of modern astrophysical research. Description is given of the relation between color, class of variability, length of period, and spectrum, mentioning the connection between color and temperature and the observed fact that fainter stars have deeper color, although the effect of color on the eye is less for faint stars. The author regrets the lack of a catalogue of star colors, apparently overlooking such catalogues as the Potsdam *Durchmusterung* and the Revised Harvard Photometry. A thorough description is given of the reported changes in the color of Sirius, α Boötis, α Ursae Majoris, 95 Herculis, and 70 Ophiuchi. A table is given

of 20 well-known variables showing a general agreement of different observers that the color grows deeper as the light decreases to minimum. As a result, the photographic range of variation is greater than the visual, which seems to contradict the statement made on page 209 that the photographic range is less than the visual. If the data given on pages 558–561 are arranged in order of color-index, additional proof is given that the ratio between photographic and visual amplitude increases as the color becomes deeper.

Descriptions are given of the changes in the color of the novae, and of their spectra, since it has become possible to observe them. No mention is made of what is perhaps the most striking change in the spectra of novae, that from absorption to emission spectra, which usually takes place about the time of maximum light.

The last chapter, xxiv, summarizes the results of the investigations on variables, and states them in the form of a few general laws dealing with length of period, shape of light-curve, color, and galactic latitude. So much information is given in a short table on page 763 that I reproduce it here.

DIFFERENCE BETWEEN SHORT- AND LONG-PERIOD VARIABLES

Period	Less than 3 Months	Greater than 4 Months
Amplitude	less than 1.5 mags.	3 mags. to 5 mags.
Irregularities	secular	periodic
Number of variables	first maximum	second maximum
Minima	sharp	flat
$(M-m)/P$	0.3 to 0.4	0.4 to 0.5
Inflections	after maximum	before maximum
Color	2C to 5C	6C to 8C
Spectra	I to II	III to IV
Galactic position	condensed	scattered

Reference is made to the second volume which deals with the physical explanation of the phenomena of variability, prepared by Father J. Stein. This is in press and will appear shortly.

This work can be obtained from the B. Herder Book Co., 17 South Broadway, St. Louis, at $13.00 for the four parts; the first three at $2.70 each and the fourth at $4.90. The weight is ten pounds.

J. A. PARKHURST

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Tables annuelles de Constantes et données numériques de Chimie, de Physique et de Technologie. Vol. IV. Années 1913, 1914, 1915, 1916. Gauthier-Villars, Cambridge University Press, University of Chicago Press. Première partie (1921). Deuxième partie (1922). 4to, pp. xxxii+626, xxxv+750, respectively. Net price for each part, $13.25; postpaid, $13.65.

The first three volumes of the annual tables of chemical and technical constants were published in 1912–1914, respectively, for the years 1910–1912. Started by an international committee under the auspices of the International Association of Academies, these volumes of tables have at once become reference works of first importance. That the international turmoil which followed these first years interfered seriously with the young organization is not surprising. Happily, however, mainly due to the energy of the general secretary, Dr. Ch. Marie, the enterprise fought its way through the difficulties and with the present double volume the gap of the years 1913–1916 inclusive has been bridged. The work of a large number of compilers is brought together here. It is divided under forty-six headings, which are listed in four languages in the Introduction. The various items included under each heading are also given there so that one can refer at once to the desired page. All the numerical values are fully explained and bibliographic references given. The reader will appreciate the policy adopted to condense the material more than has been done in the previous volumes by leaving out some uncertain values and especially by substituting graphical representations for long numerical tables.

The committee of publication is already at work on the next volume, which is to cover the data for the years 1917–1921, and urges all those who use the book to contribute to the further perfection of the tables by calling attention to omissions or errors in the previous volumes.

G. V. B.